Studying mathematics is about making connections. We strive to connect one concept to the next to tell a story. As a teacher, I try to embody this at all levels. From a broad perspective, it’s key to design a syllabus so that each topic follows seamlessly from the last, usually connecting with other earlier concepts as well, while maintaining forward momentum. This carries down to the micro level of designing a homework assignment, where students are led through examples that build on each other and become more complex.

In a typical lecture course, I believe in leading students through these connections by asking them questions. In an Elementary Calculus course, I asked my students, “Suppose we have an equation giving $y$ as a function of $x$. We’ve taken derivatives of equations like this before with respect to $x$. But as we’ll see soon, we might want to view $x$ and $y$ both as functions of another variable, say $t$, and take a derivative with respect to $t$. How can we accomplish this using what we already know?” Having recently learned the chain rule, students were able to suggest (after a bit of cognitive discourse) that we could apply the chain rule to discover implicit differentiation for themselves. When we have an open question, it’s important to look back to known techniques. And just as importantly, when we’ve solved that problem, we should look forward to see what doors to new questions have opened for us. In this case, I suggested that the not-so-innocuously named variable $t$ could stand for time, thus guiding us toward related rates. Students were thus prepared to tackle the WebWork problem set as well as to progress forward in the course.

The key to effective communication is knowing your audience. This is especially true when trying to connect with students. What about the course and its material is important to them? Why should it matter? I try to motivate each topic in a course by demonstrating either how it’s directly related to something applicable to those students, or how it serves as a tool to enable an application. In that Elementary Calculus course, where many of the students are business majors, I motivated the exponential function with both the idea of coming up with a (non-trivial) function that is its own derivative, as well as giving us a way to compute continuously compounded interest.

When I taught Mathematical Problem Solving for Elementary Teachers, my audience was significantly different: a class of hopeful education majors, who were already armed with teaching tools, but needed to learn how to use them to demonstrate basic mathematical concepts. With autonomy as the instructor for the course, I decided to let my students explore the material on their own when possible. I supplemented my own lectures by having these future teachers give demonstrations and presentations to connect back to the overarching theme of how to explain the material to elementary schoolers. To set an example, I incorporated student self-discovery of mathematical concepts into my own lectures. One class, I came in armed with stacks of triangles I had cut out of construction paper, along with scissors and tape. I explained, “Last class, we computed the areas of all sorts of rectangles. Today, I want you to discover a formula for the area of a triangle. I expect that many of you remember such a formula, though some of you may not. Either way, I want you to focus on how you can discover a formula. And once you’ve got it, see how many other ways you can arrive at the same solution.” Students who didn’t recall the formula felt accomplished deriving the formula themselves; students who did recall it found it interesting to actually show how it worked. And everyone experienced actually doing mathematics, and not being force-fed the material for its own sake. In the future, when they are called upon to explain the “why?” and the “how?” by an inquisitive student, I hope that my former students will share with them the tools they learned in my class — not only how the mathematics works, but how satisfying it is to figure it out on one’s own.
Open-ended programs are an excellent opportunity for inquiry-based learning. Indeed, while working with the Math Circle at a local elementary school and at University of Kentucky’s High School Math Day, I’ve witnessed first-hand the a-ha moments of students attempting to build polyhedra out of plastic building bits or folded paper, trying to make the angles work out just right. And when they do, it’s brilliant. But rather than bask in the glory of having successfully built a cube, it’s time already to move forward with a question like “So what else can we make?” or “Can you make any other 3-D figures out of only square faces?” or “What do you notice about the corners of the cube?” depending on the audience. But critically, in an exercise like this where students wield considerable freedom, I believe in letting them answer their own questions. In any course I believe in regularly questioning the students to check not only knowledge and comprehension, but also how we can proceed. But in this sort of atmosphere, where there may be no true syllabus, I prefer to answer questions with my own questions. For every “How do you do this?” I have a “What have you tried?” In the end, I believe the students are better off for it, as it lets them work on their own problem solving skills while still getting occasional guidance as necessary.

I’ve had success with the same technique during my time in UK’s MathExcel program. Students in MathExcel attend the same Calculus I & II lectures as other students, but they enjoy longer and more frequent workshop sessions, which are led by a graduate student with the assistance of two experienced undergraduates. Designed for students willing to put in extra time and effort to truly understand calculus, MathExcel gave me an opportunity to push students toward success. In writing worksheets for students to tackle in small groups, I would make sure that the exercises told a story and appropriately addressed the information covered in class. Problems toward the end of the worksheet would force students not only to use what they’d learned in the previous lecture, but also to actually think about how the concepts work. For example, after having students compute some basic limits of sequences, I asked them to show that the sequence of successive ratios of Fibonacci numbers approaches a number \( r \) satisfying \( r^2 = r + 1 \). Upon reaching this problem, most groups didn’t know how to start the problem, and would ask for help. “Where do we begin?” they would ask me, and I’d respond with, “What do you know?” And after they’d given me the Fibonacci recurrence relation, I’d let them work out where to go from there; usually I’d hear an excited “Oh!” from that table a minute later, as the group realized how to connect that relation and what they’d learned about limits the previous term. I look forward to future opportunities to facilitate students learning math by doing it for themselves.

I understand that not all students are so motivated and enthusiastic about doing mathematics. Both of the College Algebra classes I taught were filled with students from a wide variety of majors who were enrolled primarily to meet a general education requirement. Many of these students truly believed that for some reason they are ‘bad at math.’ Finding motivating examples in an application of mathematics for such a class is difficult, and often will only appeal to a small set of students. To combat this, I placed extra emphasis on the idea of connecting one concept to the next and focused on teaching the students to learn by doing, and that solving problems logically will aid them in whatever they end up doing, no matter how much math there appears to be in a problem at first blush. Even the self-proclaimed ‘bad at math’ student, with the right support, can learn how to overcome their negative beliefs approach problems from a mathematical perspective. While I love doing and teaching mathematics, I know that not every student will share my passion for the subject.

I remain steadfast in my efforts to show students of all backgrounds why mathematics should matter to them. Despite what they may believe, they too are capable of doing mathematics. And I intend to improve my own abilities by learning to use new strategies and technology in the classroom do facilitate the creation of new connections.