Graduate Student Combinatorics Conference

Conference Schedule

Saturday, March 28, 2009

8:00-8:45 - Check-In - CB Lobby

8:45-9:00 - Welcoming Remarks - CB 118

9:00-10:00 - Keynote Address I - CB 118

Alternating permutations

Richard Stanley

9:00-10:00 Saturday

Abstract

A permutation $a_1 a_2 \cdots a_n$ of $1, 2, \ldots, n$ is *alternating* if $a_1 > a_2 < a_3 > a_4 < \cdots$. We will survey the enumerative properties of alternating permutations. In particular, if E_n is the number of alternating permutations of $1, 2, \ldots, n$, then

$$\sum_{n \geq 0} E_n \frac{x^n}{n!} = \sec x + \tan x,$$

a famous result of Désiré André. The talk will also include some examples of where E_n occurs in other enumerative problems, the distribution of the longest alternating subsequence of a random permutation, some umbral formulas for enumerating classes of alternating permutations, and the special role of alternating permutations in the problem of counting permutations $a_1 a_2 \cdots a_n$ where we specify the set $\{i \colon a_i > a_{i+1}\}$.

10:00-10:30 - Coffee - CB Lobby

Coffee and other refreshments will be available in the Lobby.

10:30-12:00 - Morning Session A - CB 118

Distributions of Generalized Permutation Patterns over Words

Andrew Baxter

10:30 Saturday

Abstract

In 2000 Babson and Steingrímsson introduced generalized permutation patterns. The definitions were general enough to apply beyond permutations to words over N. We apply the Goulden-Jackson cluster method to generate a recursion computing the number of words of a given size which contain a given number of occurences of a generalized permutation pattern.

The Mobius Numbers of some sporadic groups

Joe Bohanon

11:00 Saturday

Abstract

For a finite group G, the Möbius number, $\mu(G)$ is defined to be $\mu_{L(G)}(1, G)$ where $L(G)$ is the subgroup lattice of G and μ is the standard Möbius function for a poset. In this talk, I will present some new methods for computing the Möbius number of several sporadic groups, including the Suzuki and Rudvalis groups. The full lattice of the Suzuki group is computed, while we are able to obtain the $\mu(Ru)$ without computing all subgroups. Many of these calculations rely on the computer system GAP.

Enumeration of the distinct shuffles of permutations

Camillia Smith Barnes

11:30 Saturday

Abstract

A shuffle of two words is a word obtained by concatenating the two original words in either order and then sliding any letters from the second word back past letters of the first word, in such a way that the letters of each original word remain spelled out in their original relative order. Examples of shuffles of the words 1234 and 5678 are, for instance, 15236784 and 51236748. In this talk, I enumerate the distinct shuffles of two permutations of any two lengths, where the permutations are written as words in the letters 1,2,3,...,m and 1,2,3,...,n, respectively.

10:30-12:00 - Morning Session B - CB 214

Differences of Augmented Staircase Skew Schur Functions

Matthew Morin

10:30 Saturday

Abstract

Determining when differences of the form $s_{D_1} - s_{D_2}$ are Schur positive, for skew diagrams D_1 and D_2, has recently received much interest. We inspect diagrams consisting of staircases with hook diagrams attached to their base. Fixing a particular staircase, we inspect the difference of skew Schur functions between all pairs of such diagrams with hooks of equal size. We give necessary and sufficient conditions for the difference to be Schur positive and display the relevant Hasse diagram.

A geometric construction of k-Schur polynomials

Cory Brunson

11:00 Saturday

Abstract

Cocycles of Schubert varieties generate the cohomology ring of a Grassmannian, which is isomorphic to a quotient of the ring of symmetric functions. Under this map, Schubert cocycles correspond to Schur polynomials. By lifting the Grassmannian to matrix space, Knutson and Miller constructed these polynomials directly from geometry, as multidegrees of matrix Schubert varieties.

Analogously, cocycles of affine Schubert varieties generate the cohomology of the affine Grassmannian. In this case the homology groups form a ring isomorphic to a subring of symmetric functions, and affine Schubert cycles correspond to k-Schur polynomials. From affine Schubert varieties we may sinuously construct matrix varieties, whose multidegrees (cohomology classes) we conjecture to be the corresponding k-Schur polynomials under Poincaré duality.

Shelling Small Polytopes

DJ Wells

11:30 Saturday

Abstract

A d-simplex can be shelled in any of the possible $(d + 1)!$ orders of the facets and each of the 2^n subsets of the facets form a shellable ball. However, if we don't label the vertices, there is only one shelling order, and only one shellable patch of each size. I will investigate the combinatorially distinct shelling orders and shellable patches of small polytopes that are not quite simplices (ones with just one "extra" vertex). We will see that these shelling orders, partial shelling orders, and shellable patches are all in bijection with some familiar objects.

12:00-1:30 - LUNCH

See the "Restaurants" sheet for a list of places within walking distance.

1:30-3:30 - Afternoon Session A - CB 118

Energy of Graphs and Digraphs

Nafiseh Jahanbakht

1:30 Saturday

Abstract

Suppose $\lambda_1, \lambda_2, \cdots, \lambda_n$ are the eigenvalues of the adjacency matrix of a graph G. The *energy* of the graph G, denoted by $\mathcal{E}(G)$, is $\mathcal{E}(G) = \sum_{i=1}^{n} |\lambda_i|$. Recently, a lot of research has been carried out on this concept. The generalization of the concept has been made for the case of digraphs. We found a new class of with relatively high energy and also the energy of some classes of digraphs and their relation with the energy of their underlying graph.

Graph Amalgamations and Hamiltonian Decompositions of Multigraphs

Mohammad Amin Bahmanian

2:00 Saturday

Abstract

A Hamiltonian decomposition is a decomposition of a regular graph into spanning cycles. Amalgamating a graph H can be thought of as taking H, partitioning its vertices, then for each element of the partition squashing the vertices to form a single vertex in the amalgamated graph G. Any edge incident with the original vertices in H are then incident with the corresponding new vertex in G, and any edge joining two vertices that are squashed together in H becomes a loop on the new vertex in G. Graph amalgamation has been proved very powerful in constructing Hamiltonian decompositions of various classes of graphs.
In this talk we describe this technique and we give a generalization of the previous results in graph amalgamation. Then we give a necessary and sufficient condition for $K(a_1, \ldots, a_p; \lambda_1, \lambda_2)$ to be Hamiltonian decomposable using this generalization, and finally we give some extensions for almost-regular graphs.

This is joint work Chris A. Rodger.

Packings in Steiner Triple Systems

Juliana Freire

2:30 Saturday

Abstract

A packing in a Steiner Triple System is a maximal set of non-overlapping edges. A packing is perfect if it uses all vertices in the case $n = 6k + 3$ or all but one vertex in the case $n = 6k + 1$. It is known that every STS with n vertices has a packing using all but at most $O\left(\sqrt{n}\ln(n)^{3/2}\right)$ vertices. A random greedy packing algorithm finds packings leaving around that many vertices unpacked. With a heuristic modification we have been able to find perfect packings in all of the test cases in every run.

Enumeration Results on the Linear Complexity of Sequences

Ramakanth Kavuluru

3:00 Saturday

Abstract

Linear feedback shift registers (LFSRs) are fast devices that are used as building blocks for generating key streams in stream ciphers. The linear complexity $L(S)$ of a sequence $S = (s_0, s_1, \cdots)$ over F_q is the length of the smallest LFSR that can generate S. Alternatively, it is also the least order of a homogeneous linear recurrence relation satisfied by S. The k-error linear complexity $L_k(S)$ of S is the smallest linear complexity obtained by making k or fewer changes in S. For cryptographic purposes it is essential that sequences have high linear complexity and high k-error linear complexity. In this talk I will present several recent results on counting functions for the number of sequences with fixed linear complexity or k-error linear complexity.

1:30-3:30 - Afternoon Session B - CB 214

Characterizing the Face Numbers of Simplicial Complexes

Jonathan Browder

1:30 Saturday

Abstract

One of the most fundamental invariants of a simplicial complex Γ is its f-vector, which counts the number of faces of Γ in each dimension. A central problems in geometric combinatorics is that of characterizing the f-vectors of various classes of simplicial complexes. The Kruskal-Katona theorem completely characterized the f-vectors of general simplicial complexes; later Stanley provided a complete characterization of f-vectors of complexes which are Cohen-Macaulay. A generalization of Stanley's theorem will be presented, motivated by the study of complexes having certain kinds of symmetry.

The parameter space of generalized permutohedra

Jeffrey Doker

2:00 Saturday

Abstract

There is an interesting type of polytope called the permutohedron. If you wiggle the facets of a permutohedron, you get what's called a generalized permutohedron. Generalized permutohedra come in many different shapes and sizes, and this all depends on an assortment of parameters. I will talk about these parameters, and about the parameter space they inhabit.

The Complete **cd**-index of Dihedral Intervals and Finite Coxeter groups.

Saúl A. Blanco

2:30

Abstract

Consider a Coxeter system (W, S) and a reflection ordering $<_T$ of the set T of reflections of (W, S). One can encode the descent distribution of the paths of Bruhat graph of the Bruhat interval $[u, v]$ with a non-homogeneous polynomial in non-commutative variables **c** and **d**. This polynomial is called the *complete* **cd***-index* of $[u, v]$. The terms of highest degree in the complete **cd**-index form the regular **cd**-index of an Eulerian poset. On the other hand, not much is known about the lower-degree terms. We describe what the lower-degree terms are in case of dihedral intervals as well as the lowest-degree terms for finite Coxeter groups.

The Fundamental Group of Balanced Simplicial Posets

Steven Klee

3:00 Saturday

Abstract

A great deal of work has been done to relate the combinatorics of a simplicial complex Δ to the dimensions of its homology groups with coefficients in a certain field $\mathbf{k}$. In contrast, very little seems to be known about the relationship between the homotopy groups of Δ and the combinatorics of Δ. The Hurewicz theorem tells us that when Δ is connected, $H_1(\Delta; Z)$ is the abelianization of $\pi_1(\Delta, *)$ so that the number of generators of $\pi_1(\Delta, *)$ is at least as large as the number of generators of $H_1(\Delta, *)$ (and hence at least as large as the dimension of $H_1(\Delta; \mathbf{k})$). We will give a new result bounding the size of a minimal generating set of $\pi_1(\Delta, *)$ in terms of combinatorial invariants of Δ for a certain class of balanced simplicial complexes and, more generally, balanced simplicial posets.

3:30-4:00 - Coffee - CB Lobby

4:00-5:30 Poster Session - CB Lobby

Betti numbers and torus embeddings

Letitia Banu

Abstract

Some results about torus embeddings and J-irreducible monoids. Starting with an irreducible representation (of "type" J) of a semisimple algebraic group G we associate a projective toric variety X(J). Renner classifies all J's such that X(J) is rationally smooth and then describes the Betti numbers of X(J) in terms of the Weyl group of G and a so-called "descent systems". We survey this work and then present a detailed example. We conclude with some suggestions for future work in this direction.

Geometric Designs, Reed-Muller Codes, and Hamadas Conjecture

David Clark

Abstract

Reed-Muller codes are a class of error-correcting codes which admit fast and simple construction and decoding. We show that these codes contain highly structured combinatorial incidence structures called designs. These codes and designs prove useful in determining the types of designs for which a conjecture of Hamada is false, as well as certain cases for which it is true. We give a survey of current results, as well as several new results concerning the truth of the conjecture.

The Association Scheme of a Cayley Graph

Jessica Ellis

Abstract

We investigate Cayley graphs whose adjacency matrices have few eigen- values. We explore the correspondence between the spectrum of a Cayley graph and the associate classes of an induced association scheme on the ver- tices of the graph. A Cayley graph is a graph which has an automorphism group acting sharply transitively on the vertices of the graph. Explicitly, given a nite group G and a subset $S \subset G$, the Cayley graph is dened with vertex set G so that two vertices g, h are adjacent if and only if there exists an element s in S such that $h = sg$. Given a Cayley graph $\Gamma(G, S)$, we construct an association scheme on the vertices so that the number of classes of the association scheme is the number of distinct eigenvalues of the graph. We are especially interested in simple graphs with few distinct eigenvalues/classes. (Included in our investigation are Cayley graphs which are also distance regular.) This poster reports on joint work with professor Ken Smith and three under- graduate students from a summer research program at Sam Houston State University, funded by the National Science Foundation and Sam Houston State University.

Matrices with Row and Column Sum 2

Janine Janoski

Abstract

Let $\mathcal{M}(n,2)$ be the number of $n \times n$ matrices with binary entries, row and column sum 2, and whose rows are in lexicographical order. Let $M(n,2) = |\mathcal{M}(n,2)|$. We will show $M(n,2)$ satisfies $M(n,2) = (n-1)M(n-1,2) - \frac{(n-1)(n-2)}{2}M(n-3,2) + (n-1)M(n-2,2)$.

We will also discuss sets of related matrices.

Joint work with Neil Calkin, Clemson University

4-cycle systems of the line graphs of complete multipartite graphs

Nidhi Sehgal

Abstract

We investigate the necessary and sufficient conditions for the existence of 4-cycle systems of the line graphs of complete multipartite graphs.

Fundamental Transversal Matroids and Lattices of Cyclic Flats

Ken Shoda

Abstract

In the axiom scheme for matroids by cyclic flats, the rank of a set X is given by $r(X) = \min\{r(Z)+|X-Z| : Z \in \mathcal{Z}(M)\}$, where $\mathcal{Z}(M)$ is the set of cyclic flats of M. Thus the set of cyclic flats with $r(X) = r(Z) + |X - Z|$ determines the rank of X. The set $R(X)$ of such cyclic flats has a number of interesting properties, some of which I explore.

Joseph Bonin and Anna de Mier showed that, given an abstract lattice L, there is a matroid M whose lattice of cyclic flats, $\mathcal{Z}(M)$, is isomorphic to L. The matroid M they construct is a fundamental transversal matroid with a fundamental basis B; each cyclic flat of M is spanned by a subset of B. I noticed that M is a fundamental transversal matroid if and only if $\mathcal{Z}(M) = R(X)$, and we can always choose X to be a fundamental basis. Thus, for given lattice L, there is a matroid M such that $L \cong \mathcal{Z}(M) = R(X)$.

Using this result, I determined all integer-valued functions ρ on a given abstract lattice L such that there is a fundamental transversal matroid M with a lattice isomorphism $\varphi : L \to \mathcal{Z}(M)$ satisfying $\rho = r\varphi$. Furthermore, I show how to find the unique ρ that minimizes the rank and cardinality of a matroid.

Major Index for 01-Fillings of Moon Polyominoes

Svetlana Poznanovik

Abstract

We propose a major index statistic on 01-fillings of moon polyominoes which, when specialized to certain shapes, reduces to the classical major index for permutations and set partitions. We consider the set $\mathbf{F}(\mathcal{M}, \mathbf{s}; A)$ of all 01-fillings of a moon polyomino $\mathcal{M}$ with given column sum $\mathbf{s}$ whose empty rows are A, and prove that this major index has the same distribution as the number of north-east chains, which are the natural extension of inversions (resp. crossings) for permutations (resp. set partitions). Hence our result generalizes the classical equidistribution results for the permutation statistics inv and maj. Two proofs are presented. The first is an algebraic one using generating functions, and the second is a bijection on 01-fillings of moon polyominoes in the spirit of Foata's second fundamental transformation on words and permutations (joint with W. Y. C. Chen, C. Yan, and A. L. B. Yang).

Classification of the additive $[12, 3.5, 8]_4$ codes, part 1

Chao Zhong

Abstract

So far what is known about the additive $[12, 3.5, 8]_4$ codes is the code discovered by A. Blokhuis & A.E. Brouwer via a computer program. In this thesis we classify the strength 1 $[12, 3.5, 8]_4$ codes by their algebraic and geometric properties.

6:00 - Conference Banquet - Student Center Small Ball Room

Appetizers will be available shortly following the poster session. Dinner is at 7:00pm.

Sunday, March 29, 2009

8:30-10:00 - Morning Session 1A - CB 118

An explicit formula for the antipode in the Hopf algebra of non-crossing partition lattices

Hillary Einziger

8:30 Sunday

Abstract

The antipode of a lattice of non-crossing partitions in the Hopf algebra of non-crossing partition lattices can be calculated though a formula that sums over all chains in the lattice. I demonstrate a surjective map from the chains in a non-crossing partition lattice to a class of polygon partitions and arrive at an explicit formula for the antipode.

Recent results on the MCP conjecture

Mirko Visontai

9:00 Sunday

Abstract

We discuss some recent progress on the Monotone Column Permanent (MCP) conjecture. We use a general method for proving real rootedness of a univariate polynomial, namely by showing that a corresponding multivariate polynomial is stable. Recent connections between stability of polynomials and the strong Rayleigh property revealed by Brändén allows for a computationally feasible check of stability for multi-affine polynomials. Using this method we obtain a simpler proof for the $n = 3$ case of the MCP conjecture, and a new proof for the $n = 4$ case. This work is joint with James Haglund.

Hamiltonian Cycle Decompositions of 6-Regular Cayley Graphs on Finite Abelian Groups

Erik Westlund

9:30 Sunday

Abstract

It is known that every connected Cayley graph $\Gamma = (A, S)$ on a finite abelian group A, with generating set S, is Hamiltonian. Alspach has conjectured that if Γ is $2k$-regular, then Γ has a complete Hamiltonian cycle decomposition, i.e., a partition of the edge set into k Hamiltonian cycles. We discuss recent progress on this conjecture when $k = 3$ by examining a quotient graph, $(A/\langle s\rangle, \varphi(S))$, for some $s \in S$ with canonical map $\varphi : A \to A/\langle s\rangle$. We show in some cases, Hamiltonian decompositions are obtained by viewing Γ as a layered pseudo-cartesian product of cycles and performing edge color-switching configurations.

8:30-10:00 - Morning Session 1B - CB 214

Free and non-free multiplicities of hyperplane arrangement

Mehdi Garrousian

8:30 Sunday

Abstract

An arrangement of hyperplanes consists of a finite number of hyperplanes in finite vector space. To this object one can associate many algebraic, topological and combinatorial objects, each revealing some of its properties from a certain point of view.

The focus of the talk is going to be on the free/non-free arrangements which naturally generalize to multiarrangements. I will explain the significance of the free arrangements and show how to obtain new free multiarrangements out of a given free multiarrangement. It has been unknown for a long time whether freeness is combinatorial; however there are results of nice combinatorial flavor which motivate you to look for more. The aim of my talk will be to invite the audience to share the same feeling.

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k-Parabolic Subspace Arrangements

Jacob White

9:00 Sunday

Abstract

Given a finite real reflection group W of rank n and a positive integer $k \leq n$, we define a subspace arrangement called the k-parabolic arrangement. When W is of type A, B, or D we recover the well-known k-equal arrangements of the corresponding type. In this talk, we study some of the properties of k-parabolic arrangements. One of our results is a nice presentation for the fundamental group of the complement of the 3-parabolic arrangement. This presentation generalizes previously known results due to Khovanov. The proof uses combinatorial techniques related to the notion of discrete homotopy theory. We give a conjecture for extending other known results about k-equal arrangements to the collection of k-parabolic arrangements. This is joint work with Hélène Barcelo and Christopher Severs.

Tropical Oriented Matroids

Kristen Freeman

9:30 Sunday

Abstract

I will define the notion of a tropical oriented matroid (TOM), an object which captures the combinatorial structure of a tropical hyperplane arrangement, and shares several of the properties of ordinary oriented matroids. I will show how a TOM determines a subdivision of a product of two simplices, and provide strong evidence that this correspondence is a bijection.

Most of this work was done by Federico Ardila and Mike Develin, but I will provide additional evidence of the bijection between a TOM and a subdivision of a product of two simplices.

10:00-10:30 - Coffee - CB Lobby

10:30-11:30 - Morning Session 2A - CB 118

A Combinatorial Proof of a Recent Theorem of Andrews'

Kagan Kursungoz

10:30 Sunday

Abstract

In a recent paper, Andrews made an extensive study of parity considerations in partitions, and he obtained many new results. We will focus on the generating function he gave for partitions into distinct non-consecutive parts (partitions counted by the series side of the first Rogers-Ramanujan Identity) classified according to the number of even parts in them. We will give a combinatorial construction of the function, which is originally derived analytically.

Enumeration of minimum genus embeddings of $K_{3,n}$

Adam Weaver

11:00 Sunday

Abstract

Ringel determined the minimum orientable and non-orientable genus for all complete bipartite graphs. For $K_{3,n}$ the orientable genus is $\lceil \frac{n-2}{4} \rceil$ and the nonorientable genus is $\lceil \frac{n-2}{2} \rceil$. Euler's formula implies that nearly all of the faces in such an embedding must be 4-cycles. We show that these embeddings can be represented by certain edge colorings of n-vertex cubic graphs. Using this correspondence, we provide enumerative results for all minimum genus embeddings of $K_{3,n}$.

10:30-11:30 - Morning Session 2B - CB 214

Nim on Graphs

Lindsay Merchant

10:30 Sunday

Abstract

The game of Nim is a relatively simple two-player take away game. Its solution has been known for quite some time and relies on the use of Grundy numbers. When extending the game of Nim to graphs by assigning weights to different edges, the problem of finding the solution becomes exceedingly more difficult. Masahiko Fukuyama (2003) showed which player has a winning strategy for certain bipartite graphs. He also found the Grundy number for Nim on trees and Nim on cycles. This talk presents new results for Nim on K_n for small values of n, solving the winning strategy problem completely for these graphs under any weight assignment. We also discuss the relative importance of Grundy numbers for Nim on graphs.

A bijective proof of Bressoud's Conjecture related to the Rogers-Ramanujan Identities

Shishuo Fu

11:00 Sunday

Abstract

The Rogers-Ramanujan Identities have many natural and significant generalizations. The generalization presented here was first studied by D. Bressoud, by considering the partitions he named as footed partition. A bijective proof to his conjecture is given and some examples will be discussed in the end.

11:30-12:30 - KEYNOTE ADDRESS II - CB 118

The Erdős-Moser conjecture

Richard Stanley

11:30-12:30 Sunday

Abstract

Given a finite set S of real numbers and a real number α, let $f(S,\alpha)$ be the number of subsets of S whose elements sum to α. For instance, $f(\{1,2,3,4,5,6\},10) = 5$. Given n, what is the maximum possible value of $f(S,\alpha)$? Erdős and Moser conjectured that for $n = 2m+1$ the maximum is achieved by taking $S = \{-m, -m+1, \ldots, m\}$ and $\alpha = 0$. (There is a similar conjecture for n even.) We will indicate how tools from linear algebra and partially ordered sets can be used to prove this conjecture. At present no purely combinatorial proof is known. This talk is independent from the first talk.

12:45 - Closing Remarks - CB 118