1. (Topologist’s sine curve) Let \( \Gamma \subseteq \mathbb{R}^2 \) be the graph of \( \sin(1/x) \) for \( 0 < x \leq 1/\pi \). Show that the closure \( \overline{\Gamma} \) is connected but not path-connected, locally connected, or locally path-connected.

2. Show that \( \mathbb{R}_{\text{cocountable}} \) is connected and locally connected but not path-connected or locally path-connected.

3. For any space \( X \), define the cone on \( X \) to be
\[
CX = (X \times I)/(X \times \{1\}).
\]
Show that \( CX \) is path-connected (no assumptions on \( X \)).

4. Show that \( \mathbb{R}_{\text{cofinite}} \) is compact.

5. Show that if \( X \) is a metric space and \( A \subseteq X \) is compact, then \( A \) is closed and bounded (contained in a single ball of finite radius).

6. (a) Show that if \( X \) is compact and \( Z \) is any space, then the projection \( p_Z : X \times Z \longrightarrow Z \) is closed.
   (b) (*) Show that the converse to (a) also holds. That is, if \( p_Z \) is closed for every \( Z \), then \( X \) is compact.