1. (Cantor set) Let \( A_0 = I = [0, 1] \). Define \( A_1 = A_0 \setminus \left( \left( \frac{1}{3}, \frac{2}{3} \right) \cup \left( \frac{7}{9}, \frac{8}{9} \right) \right) \). Similarly, define \( A_2 \) by removing the middle thirds of the intervals in \( A_1 \):

\[
A_2 = A_1 \setminus \left( \left( \frac{1}{9}, \frac{2}{9} \right) \cup \left( \frac{7}{9}, \frac{8}{9} \right) \right).
\]

In general, given \( A_n \) constructed in this way, we define \( A_{n+1} \) by removing the middle thirds of all intervals in \( A_n \). Define the Cantor set to be

\[
C = \bigcap_n A_n \subseteq [0, 1].
\]

(a) Show that \( C \) is compact (without using part (d)).
(b) Show that any compact, locally connected space has finitely many components. Conclude that \( C \) is not locally connected.
(c) Show that \( C \) is totally disconnected (every connected component is a singleton).
(d) Let \( D = \{0, 2\} \) with the discrete topology. Show that \( C \cong \prod D \). (Hint: instead of binary expansions, think about ternary expansions of numbers in \([0, 1]\).)

2. Let \( X \) be Hausdorff, and suppose that \( C, D \subseteq X \) are disjoint compact subsets. Show that there are disjoint open sets \( U, V \subseteq X \) with \( C \subseteq U \) and \( D \subseteq V \).

3. (Stereographic Projection) Let \( N = (0, \ldots, 0, 1) \in S^n \) be the North Pole. Define a homeomorphism \( S^n \setminus \{N\} \cong \mathbb{R}^n \) as follows. For each \( x \neq N \in S^n \), consider the ray starting at \( N \) and passing through \( x \). This meets the equatorial hyperplane (defined by \( x_{n+1} = 0 \)) in a point, which we call \( p(x) \).

(a) Determine a formula for \( p \) and show that it gives a homeomorphism.
(b) Conclude that the one-point compactification of \( \mathbb{R}^n \) is \( S^n \).

4. Show that if \( Z \) is locally compact Hausdorff and \( q : X \rightarrow Y \) is a quotient, then

\[
q \times \text{id}_Z : X \times Z \rightarrow Y \times Z
\]

is a quotient.