# Math 551 - Topology I
# Homework 1
# Fall 2014

1. Show that the max metric on $\mathbb{R}^2$ is a metric. Recall that the max metric is defined by

$$d(\mathbf{x}, \mathbf{y}) = \max\{|x_1 - y_1|, |x_2 - y_2|\}.$$

2. (a) Let $X$ be any set. Define a metric on $X$ by

$$d(x, y) = \begin{cases} 0 & x = y \\ 1 & x \neq y. \end{cases}$$

   Show that this is indeed a metric. This is called the discrete metric.

   (b) Which sets are open in the discrete metric?

   (c) Suppose that $X$ is a discrete metric space and that $Y$ is any metric space. Show that any function $f : X \longrightarrow Y$ is automatically continuous.

   (d) ($\star$) Suppose that $Y$ is a discrete metric space. Show that the only continuous functions $\mathbb{R} \longrightarrow Y$ are the constant functions.

3. Show that $U \subseteq \mathbb{R}^2$ is open in the max metric if and only if it is open in the standard metric.

4. Let $f : X \longrightarrow Y$ be a function between metric spaces. Show that $f$ is continuous if and only if

   For every convergent sequence $(x_n) \to x$ in $X$, the sequence $(f(x_n))$ converges to $f(x)$ in $Y$.

5. Show that the intersection of two open sets in a metric space is always open.