1. A map $f : X \to Y$ is said to be proper if, for any compact subset $K \subseteq Y$, the preimage $f^{-1}(K) \subseteq X$ is compact.

   (a) Show that if $X$ is compact and $Y$ is Hausdorff, then any continuous $f : X \to Y$ is automatically proper.

   (b) Let $X$ and $Y$ be locally compact and Hausdorff. Show that a continuous map $f : X \to Y$ is proper if and only if it extends to a continuous map $\hat{f} : \hat{X} \to \hat{Y}$ with $\hat{f}(\infty_X) = \infty_Y$.

2. Show that if $X$ is normal and $A \subseteq X$ is closed, then $A$ is normal.

3. Suppose that $X$ is normal and connected. Show that if $X$ contains more than one point, it must be uncountable. (Hint: Use the Urysohn Lemma.)