Math 654 - Algebraic Topology
Homework 10
Fall 2015

1. (a) Show that free abelian groups are **flat**, meaning that if F is free abelian and

$$0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$$

is short exact, then so is

$$0 \longrightarrow F \otimes A \longrightarrow F \otimes B \longrightarrow F \otimes C \longrightarrow 0$$

   (b) Show that if $\alpha : M \longrightarrow N$ is a homomorphism of abelian groups and F is free abelian, then

$$F \otimes \ker(\alpha) \cong \ker(F \otimes \alpha), \qquad F \otimes \mathrm{im}(\alpha) \cong \mathrm{im}(F \otimes \alpha), \qquad F \otimes \mathrm{coker}(\alpha) \cong \mathrm{coker}(F \otimes \alpha).$$

   (c) Show that if C_* is a chain complex of abelian groups and F is a free abelian group, then $F \otimes \mathrm{H}_n(C_*) \cong \mathrm{H}_n(F \otimes C_*)$.

   (d) Show that if $q : C_* \longrightarrow D_*$ is a quasi-isomorphism, then so is $F \otimes C_* \xrightarrow{F \otimes q} F \otimes D_*$.

2. Show that in item (d) above, F can be replaced by a chain complex of free abelian groups. That is, show that if $q : C_* \longrightarrow D_*$ is a quasi-isomorphism, then so is the homomorphism $F_* \otimes C_* \xrightarrow{F_* \otimes q} F_* \otimes D_*$.

3. In this problem, A denotes a finitely generated abelian group.

   (a) Compute $\mathrm{Tor}(\mathbb{Z}, A)$.

   (b) Compute $\mathrm{Tor}(\mathbb{Z}/n, A)$.

   (c) Compute $\mathrm{Tor}(\mathbb{Q}, A)$.

   (d) Compute $\mathrm{Tor}(\mathbb{Q}/\mathbb{Z}, A)$.