1. Let
\[ 0 \to A \to B \to C \to 0 \]
between abelian groups and \( D \) any abelian group.

(a) Show that the following sequence is exact:
\[ 0 \to \text{Hom}(D, A) \to \text{Hom}(D, B) \to \text{Hom}(D, C). \]

(b) Show that the following sequence is exact:
\[ 0 \to \text{Hom}(C, D) \to \text{Hom}(B, D) \to \text{Hom}(A, D). \]

2. (Ext and extensions). An **extension** of \( A \) by \( M \) is a short exact sequence
\[ 0 \to M \to E \to A \to 0. \]

An equivalence of extensions \( E \sim E' \) is a homomorphism \( E \to E' \) making
\[ 0 \to M \to E \to A \to 0 \]
commute. (Note that \( \rho \) is automatically an isomorphism by the 5-lemma.) Denote by \( \text{Exten}(A, M) \) the set of equivalence classes of extensions of \( A \) by \( M \).

(a) Construct a function \( \Phi : \text{Ext}(A, M) \to \text{Exten}(A, M) \) as follows. Starting from a class \( \alpha \in \text{Ext}(A, M) \) and a resolution \( F_\ast \) of \( A \), pick a representative \( F_1 \to M \) of \( \alpha \). Then consider the diagram
\[
\begin{array}{ccc}
0 & \to & F_1 \\
\downarrow f_1 & & \downarrow \\
0 & \to & M \\
\end{array}
\]
\[
\begin{array}{ccc}
& & E \\
\rho & \nearrow & \\
& & A \\
\end{array}
\]

(b) Construct a function \( \Lambda : \text{Exten}(A, M) \to \text{Ext}(A, M) \) as follows. Given an extension \( M \to E \to A \), consider the 6-term exact sequence arising from the functor \( \text{Hom}(\_, M) \).

(c) Show that \( \Phi \) and \( \Lambda \) are inverse to each other.

3. An abelian group \( M \) is said to be \( p \)-divisible if \( M \to M \) (multiplication by \( p \)) is surjective. What does problem 2 tell you about extensions of \( \mathbb{Z}/p \) by a \( p \)-divisible abelian group \( M \)?