1. Given the $\Delta$-complex structure on the Klein bottle $K$ described in class, compute the homology groups $H^\Delta_*(K)$.

2. Let $X$ be obtained from a simplex $\Delta^2$ by identifying the three vertices to a single point. Compute the homology groups $H^\Delta_*(X)$.

3. Build $S^3$ as a $\Delta$-complex, and use this to compute $H^\Delta_*(S^3)$.

4. Consider a ball $B^3$, obtained by gluing together three $\Delta^3$’s along the edge $\{2, 3\}$ as in the picture to the right. Let $X$ be the space obtained by the gluings $e_1 \sim f_2$, $e_2 \sim f_3$, and $e_3 \sim f_1$. In the picture, $e_3$ and $f_3$ are the “back” faces. Compute $H_*(X)$. (Hint: you should find that $H_1 \cong \mathbb{Z}/3$.)