1. A chain map that induces an isomorphism in homology is called a **quasi-isomorphism**. We showed in class that any chain homotopy equivalence is a quasi-isomorphism. Give an example of a quasi-isomorphism of chain complexes which is not a chain homotopy equivalence.

2. Show that the chain complex $C_\Delta^* \left( \mathbb{R}P^2 \right)$ described in class (on 9-9-15) is chain homotopy equivalent to the complex $\mathbb{Z} \xrightarrow{2} \mathbb{Z} \xrightarrow{0} \mathbb{Z}$.

3. An **additive functor** $F : \text{AbGp} \rightarrow \text{AbGp}$ is a functor such that each function

$$F : \text{Hom}(A, B) \rightarrow \text{Hom}(FA, FB)$$

is a homomorphism. Show that if $F$ is additive, then

$$F(0) = 0 \quad \text{and} \quad F(A \oplus B) \cong F(A) \oplus F(B).$$

4. Recall that a **short exact sequence** is a sequence of homomorphisms

$$0 \rightarrow A \xrightarrow{i} B \xrightarrow{p} C \rightarrow 0$$

which is exact (has trivial homology) at each spot.

(a) A short exact sequence is called **split exact** if $B \cong A \oplus C$. Show that the following are equivalent for the (solid arrow) sequence:

$$0 \longrightarrow A \xrightarrow{i} B \xrightarrow{p} C \longrightarrow 0$$

i. The sequence is split exact

ii. There exists a map $s$ such that $p \circ s = \text{id}_C$

iii. There exists a map $r$ such that $r \circ i = \text{id}_A$

(b) Consider the functor $\mathbb{Z}/2\mathbb{Z} \otimes - : \text{AbGp} \rightarrow \text{AbGp}$. By applying this to the short exact sequence

$$0 \rightarrow \mathbb{Z} \xrightarrow{2} \mathbb{Z} \rightarrow \mathbb{Z}/2\mathbb{Z} \rightarrow 0,$$

show that this functor does not preserve short exact sequences.

(c) Show, on the other hand, that any additive functor $F : \text{AbGp} \rightarrow \text{AbGp}$ must preserve split exact sequences.