Math 654 - Algebraic Topology
Homework 8
Fall 2015

1. (Moore space) Let A be an abelian group, and suppose given a short exact sequence

$$0 \longrightarrow F_2 \longrightarrow F_1 \longrightarrow A \longrightarrow 0,$$

in which F_1 and F_2 are both free abelian groups. For any $n \geq 1$, construct a space X whose only nonzero reduced homology group is $\mathrm{H}_n(X) \cong A$.

2. Let $A_1, A_2, \ldots$ be a sequence of abelian groups. Use the previous problem to construct a space X such that $\mathrm{H}_n(X) \cong A_n$ for all $n \geq 1$.

3. (a) Let X and Y be CW complexes. Given an n-cell e^n_α of X and a k-cell e^k_β of Y, construct a map

$$\varphi_{\alpha,\beta} : S^{n+k-1} \longrightarrow (X^n \times Y^{k-1}) \cup (X^{n-1} \times Y^k)$$

that extends to

$$\Phi_{\alpha,\beta} : D^{n+k} \cong D^n \times D^k \xrightarrow{\Phi_\alpha \times \Phi_\beta} X^n \times Y^k.$$

(Hint: It may help to use the model $S^{n+k-1} = \partial I^{n+k}$.) This construction can be used to provide $X \times Y$ with a CW structure in which cells correspond to pairs of cells in X and Y, respectively.

(b) Starting from the minimal CW structure on S^1, describe the CW structure on the torus $T^2 = S^1 \times S^1$ resulting from the above construction.

(c) Show that if X and Y are finite CW complexes, then $\chi(X \times Y) = \chi(X)\chi(Y)$.