Math 654 - Algebraic Topology
Homework 11
Fall 2016

1. (a) Let $X$ and $Y$ be CW complexes. Given an $n$-cell $e^n_\alpha$ of $X$ and a $k$-cell $e^k_\beta$ of $Y$, construct a map

$$\varphi_{\alpha,\beta} : S^{n+k-1} \rightarrow (X^n \times Y^{k-1}) \cup (X^{n-1} \times Y^k)$$

that extends to

$$\Phi_{\alpha,\beta} : D^{n+k} \cong D^n \times D^k \xrightarrow{\phi_{\alpha} \times \phi_{\beta}} X^n \times Y^k.$$  

(Hint: It may help to use the model $S^{n+k-1} = \partial I^{n+k}$.) This construction can be used to provide $X \times Y$ with a CW structure in which cells correspond to pairs of cells in $X$ and $Y$, respectively.

(b) Starting from the minimal CW structure on $S^1$, describe the CW structure on the torus $T^2 = S^1 \times S^1$ resulting from the above construction.

(c) Show that if $X$ and $Y$ are finite CW complexes, then $\chi(X \times Y) = \chi(X)\chi(Y)$.

2. Use the above to write down the cellular chain complex for $X = \mathbb{RP}^2 \times \mathbb{RP}^2$, and use this to compute the homology with $\mathbb{Z}$ and $\mathbb{F}_2$ coefficients.