1. A chain map $f_* : C_* \to D_*$ that induces an isomorphism in homology is called a \textbf{quasi-isomorphism}. We showed in class that any chain homotopy equivalence is a quasi-isomorphism. Give an example of a quasi-isomorphism of chain complexes which is not a chain homotopy equivalence.

2. Show that the chain complex $C_\Delta^\Lambda(\mathbb{R}P^2)$ described in class (on 9-12-16) is chain homotopy equivalent to the complex $\mathbb{Z} \xrightarrow{2} \mathbb{Z} \to \mathbb{Z}$.

3. Recall that a \textbf{short exact sequence} is a sequence of homomorphisms

$$0 \to A \xrightarrow{i} B \xrightarrow{p} C \to 0$$

which is exact (has trivial homology) at each spot. A short exact sequence is called \textbf{split exact} if $B \cong A \oplus C$. Show that the following are equivalent for the (solid arrow) sequence:

$$0 \longrightarrow A \xrightarrow{i} B \xrightarrow{p} C \xrightarrow{r} 0$$

(a) The sequence is split exact

(b) There exists a homomorphism $s$ such that $p \circ s = \text{id}_C$ ($s$ is called a splitting)

(c) There exists a homomorphism $r$ such that $r \circ i = \text{id}_A$ ($r$ is called a retraction or splitting)