1. (a) Let $X$ be any set. Define the discrete metric on $X$ by

$$d(x, y) = \begin{cases} 
0 & x = y \\
1 & x \neq y.
\end{cases}$$

Show that this is indeed a metric.

(b) Which sets are open in the discrete metric?

(c) Suppose that $X$ is a discrete metric space and that $Y$ is any metric space. Show that any function $f : X \rightarrow Y$ is automatically continuous.

(d) $(\star)$ Suppose that $Y$ is a discrete metric space. Show that the only continuous functions $\mathbb{R} \rightarrow Y$ are the constant functions.

2. Show that $U \subseteq \mathbb{R}^2$ is open in the max metric if and only if it is open in the standard (Euclidean) metric.

3. (a) Show that the wheel metric on $\mathbb{R}^2$ is a metric. Recall that the wheel metric is defined by

$$d(x, y) = \begin{cases} 
d_{Euc}(x, y) & \text{if } x \text{ and } y \text{ lie on a common line through } 0 \\
d_{Euc}(x, 0) + d_{Euc}(y, 0) & \text{else}
\end{cases}$$

(b) Show that every Euclidean open set in $\mathbb{R}^2$ is also open with respect to the wheel metric, but give an example to show that the converse is not true.

4. Let $f : X \rightarrow Y$ be a function between metric spaces. Show that $f$ is continuous if and only if

For every convergent sequence $(x_n) \rightarrow x$ in $X$, the sequence $(f(x_n))$ converges to $f(x)$ in $Y$.

5. Show that the intersection of two open sets in a metric space is always open.