1. Let \((X, d)\) be a metric space and consider \(X \times X\), equipped with the max metric. Show that \(d : X \times X \to \mathbb{R}\) is continuous.

2. Let \(X, Y, \) and \(Z\) be metric spaces. Show that if \(\hat{\varphi} : X \to C(Y, Z)\) is continuous then \(\varphi : X \times Y \to Z\) is continuous. (The other implication is not always true.)

3. (a) (The cofinite topology) Let \(X\) be an infinite set. Define a nonempty subset \(U \subseteq X\) to be open if \(X \setminus U\) is finite. Show that this defines a topology on \(X\).

   (b) (The cocountable topology) Let \(X\) be an infinite set. Define a nonempty subset \(U \subseteq X\) to be open if \(X \setminus U\) is countable. Show that this defines a topology on \(X\).

   (c) In the case \(X = \mathbb{R}\), how do these relate to each other and to the usual topology?

4. (a) (Generic point topology) Let \(X\) be a set, and fix a special point \(x_0 \in X\). Declare a nonempty subset \(U \subseteq X\) to be open in \(X\) if and only if \(x_0 \in U\). Show that this gives a topology on \(X\).

   (b) (Excluded point topology) Let \(X\) be a set, and fix a special point \(x_0 \in X\). Declare a proper subset \(U \subset X\) to be open if and only if \(x_0 \notin U\). Show that this gives a topology on \(X\).

5. (Vertical interval topology) Define \(B\) to be the collection of vertical open intervals in \(\mathbb{R}^2\), namely, sets of the form

   \[
   \{a\} \times (b, c) = \{(a, x) \mid b < x < c\}
   \]

   for fixed \(a, b, c\). (Beware that in the left-hand side of the equation, \((b, c)\) means the open interval in \(\mathbb{R}\), whereas in the right-hand side, \((a, x)\) means the point of \(\mathbb{R}^2\).) Show that this defines a basis for a topology on \(\mathbb{R}^2\). How is this topology related to the standard topology?