Math 551 - Topology I
Homework 2
Fall 2017

1. Let (X, d) be a metric space and consider $X \times X$, equipped with the max metric. Show that $d : X \times X \longrightarrow \mathbb{R}$ is continuous.

2. Let X, Y, and Z be metric spaces. Show that if $\widehat{\varphi} : X \longrightarrow \mathcal{C}(Y, Z)$ is continuous then $\varphi : X \times Y \longrightarrow Z$ is continuous. (The other implication is not always true.)

3. (a) (The cofinite topology) Let X be an infinite set. Define a nonempty subset $U \subseteq X$ to be open if $X \setminus U$ is finite. Show that this defines a topology on X.
   (b) (The cocountable topology) Let X be an infinite set. Define a nonempty subset $U \subseteq X$ to be open if $X \setminus U$ is countable. Show that this defines a topology on X.
   (c) In the case $X = \mathbb{R}$, how do these relate to each other and to the usual topology ?

4. (a) (Generic point topology) Let X be a set, and fix a special point $x_0 \in X$. Declare a nonempty subset $U \subseteq X$ to be open in X if and only if $x_0 \in U$. Show that this gives a topology on X.
   (b) (Excluded point topology) Let X be a set, and fix a special point $x_0 \in X$. Declare a proper subset $U \subset X$ to be open if and only if $x_0 \notin U$. Show that this gives a topology on X.

5. (Vertical interval topology) Define $\mathcal{B}$ to be the collection of vertical open intervals in $\mathbb{R}^2$, namely, sets of the form

$$\{a\} \times (b, c) = \{(a, x) \mid b < x < c\}$$

for fixed a, b, c. (Beware that in the left-hand side of the equation, (b, c) means the open interval in $\mathbb{R}$, whereas in the right-hand side, (a, x) means the point of $\mathbb{R}^2$.) Show that this defines a basis for a topology on $\mathbb{R}^2$. How is this topology related to the standard topology?