Math 551 - Topology I
Homework 5
Fall 2017

1. Show that a space X is Hausdorff if and only if the diagonal subset

$$\Delta(X) = \{(x, x)\} \subseteq X \times X$$

is closed.

2. (a) Show that for any two spaces X and Y, the projection map $p_X : X \times Y \longrightarrow X$ is *open*.
   (b) Find an example to show that the projection p_X need not be closed.

3. Recall that if A is a set and we have $X_\alpha = X$ for each $\alpha \in A$, then the product $\prod_\alpha X_\alpha$ can be identified with the collection of functions $f : A \longrightarrow X$. Consider this as a space with the product topology. Show that $(f_n) \to f$ in this topology if and only if the functions converge to f *pointwise*.

4. Let $\mathcal{Z} \subseteq \mathbb{R}^{\mathbb{N}}$ be the subset consisting of sequences which are eventually zero (in other words, only finitely many of the entries are nonzero). Find the closure of $\mathcal{Z}$ in $\mathbb{R}^{\mathbb{N}}$ under the box and product topologies

5. Let $A_j \subseteq X_j$ be a subspace for each $j \in J$. Show that the subspace topology on

$$\prod_{j \in J} A_j \subseteq \prod_{j \in J} X_j$$

coincides with the product topology (here $\prod_{j \in J} X_j$ is equipped with the product topology).