1. Suppose that \((g, x) \mapsto g \cdot x\) is a left action of \(G\) on \(X\). Show that the assignment \((x, g) \mapsto g^{-1} \cdot x\) defines a right action of \(G\) on \(X\).

2. Let \(X\) be a set. For any natural number \(n\), let \(\Sigma_n\) denote the symmetric group on \(n\) letters.
   (a) Show that the assignment \((x_i) \mapsto (x_{\sigma(i)})\) defines a right action of \(\Sigma_n\) on \(X^n\). (Hint: One way to think about \(X^n\) is as the set of functions \(n \rightarrow X\), where \(n = \{1, \ldots, n\}\).)
   (b) Describe the quotient \(X^n \rightarrow X^n / \Sigma_n\)
   (c) Let \(C_n \leq \Sigma_n\) be the cyclic subgroup of size \(n\) generated by the \(n\)-cycle \((1\ 2\ \cdots\ n)\). Describe the quotient \(X^n \rightarrow X^n / C_n\).

3. Let \(G\) be a topological group and \(H \leq G\) a subgroup. Show that the closure \(\overline{H} \subseteq G\) is a subgroup.

4. Let \(G\) be a topological group acting on the space \(X\). Show that the quotient map \(X \rightarrow X/G\) is an open map.

5. (a) Show that the action of \(Gl_n(\mathbb{R})\) on \(\mathbb{R}^n\) discussed in class restricts to an action of the orthogonal group \(O(n)\) on \(S^{n-1}\).
   (b) Show that the orbit space of \(O(n)\) acting on \(\mathbb{R}^n\) is \([0, \infty)\), equipped with its standard metric topology.