1. In this problem, you will give a CW structure for $S^1 \times S^2$.

(a) Show that $\partial I^3 = (\partial I^1 \times I^2) \cup (I^1 \times \partial I^2)$ in $I^3$.

(b) Consider $S^1$ and $S^2$ with their minimal CW structures, each having two cells. Build a 3-dimensional CW complex $X$ as follows. Start with a single 0-cell, and attach both a 1-cell and a 2-cell to this point. The result is $S^1 \vee S^2$. Now attach a 3-cell, using (a) to help you define the attaching map.

(c) Find a homeomorphism $X \cong \sim S^1 \times S^2$. (Since $X$ is compact and $S^1 \times S^2$ is Hausdorff, it suffices to find a continuous bijection.)

2. The Möbius band is the quotient $M = I^2 / \sim$, where $(0, t) \sim (1, 1 - t)$. Find a CW structure on $M$.

3. The Klein bottle is the quotient $K = I^2 / \sim$ where $(0, t) \sim (1, 1 - t)$ and $(x, 0) \sim (x, 1)$. Find a CW structure on $K$.

4. For spaces $X$ and $Y$, let $[X, Y]$ denote the set of homotopy classes of maps $X \to Y$.

(a) Show that if $Y$ is contractible then $[X, Y]$ contains a single element.

(b) Show that if $X$ is contractible and $Y$ is path-connected (and nonempty), then $[X, Y]$ contains a single element.

(c) Show more generally that if $X$ is contractible, then $[X, Y]$ is in bijective correspondence with the path-components of $Y$. 