Math 751 - Vector Bundles
Worksheet 3
Fall 2018

1. Let $E$ and $E'$ be vector bundles over $X$.
   
   (a) Show that $E \oplus E'$ is a product in the category $\text{Vect}_\mathbb{R}(X)$ of vector bundles over $X$.
   
   (b) Show that $E \oplus E'$ is a coproduct in the category $\text{Vect}_\mathbb{R}(X)$ of vector bundles over $X$.

2. Recall from HW1 that a bundle can be specified by its transition functions
   \[
   \{g_{U,V} : U \cap V \to \text{GL}_n(\mathbb{R})\},
   \]
   which relate to the composition
   \[
   (U \cap V) \times \mathbb{R}^n \xleftarrow{\varphi_U^{-1}} p^{-1}(U \cap V) \xrightarrow{\varphi_V} (U \cap V) \times \mathbb{R}^n
   \]
   according to the formula
   \[
   \varphi_V \varphi_U^{-1}(x, v) = (x, g_{U,V}(v)).
   \]
   Let $E$ and $E'$ be vector bundles over $X$ of rank $n$ and $n'$, with transition functions $\{g_{U,V}\}$ and $\{g'_{U,V}\}$.
   
   (a) Find the transition functions for $E \oplus E'$.
   
   (b) Find the transition functions for $E \otimes E'$.
   
   (c) Find the transition functions for $E^*$.
   
   (d) Find the transition functions for $\text{Hom}(E, E')$. (Hint: recall that $\text{Hom}(E, E') \cong E^* \otimes E'$.)