Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 13

1. Prove Lemma 2.1.75. That is, let C be a G-coefficient system. Then show that

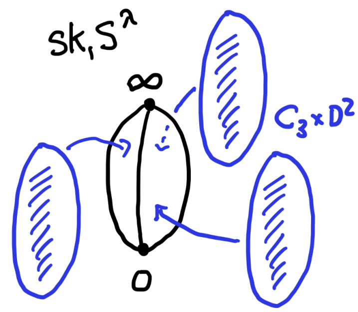
   (a) $C \otimes \underline{\mathbb{Z}} \cong C(e)/G$, (by $\underline{\mathbb{Z}}$ we mean the constant $\mathcal{O}_G$-module, whose transfer maps are all the identity)

   (b) $C \otimes \mathrm{Inf}^G_{G/G}(\mathbb{Z}) \cong C(G)$, and

   (c) $C \otimes \uparrow^G_e (\mathbb{Z}) \cong C(e)$.

2. Let C be a coefficient system for C_4. Then determine $C \otimes \mathrm{Ind}^{C_4}_{C_4/C_2}\underline{\mathbb{Z}}$. (Here $\underline{\mathbb{Z}}$ means the same as in 1(a) above.

3. Recall from Example 2.1.30 that for $G = C_3$ and λ the 2-dimensional rotation representation of C_3, we have a cell structure on S^λ with two fixed 0-cells and a single free 1-cell and single free 2-cell. We can express this as two cofiber sequences



$$C_3 \times S^0 \longrightarrow S^0 \longrightarrow S^{\lambda/2}, \qquad C_2 \times S^1 \longrightarrow S^{\lambda/2} \longrightarrow S^\lambda,$$

   where $S^{\lambda/2}$ is nonsense notation for the egg-beater displayed in black to the right.

   (a) Show that this gives a cell structure on the orbits S^λ/C_3 with contractible 1-skeleton and 2-skeleton S^2.

   (b) What does this tell you about the Bredon cohomlogy ring $\mathrm{H}^*_{C_3}(S^\lambda; \underline{\mathbb{F}_3})$?

   (c) Describe the homotopy type of the Borel construction $EC_3 \times_{C_3} S^{\lambda/2}$ on the egg-beater. Your description can be in terms of the classifying space BC_3. Deduce the Borel cohomology ring $\mathrm{H}^*_{hC_3}(S^{\lambda/2}; \mathbb{F}_3)$.

   (d) Use the cofiber sequence

$$S^1 \simeq EC_3 \times_{C_3} (C_3 \times S^1) \longrightarrow EC \times_{C_3} S^{\lambda/2} \longrightarrow EC \times_{C_3} S^\lambda$$

   to deduce the Borel cohomology ring $\mathrm{H}^*_{hC_3}(S^\lambda; \mathbb{F}_3)$.