Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 4

1. Consider a C_2-Mackey of the form $\begin{tikzcd} \mathbb{Z} \arrow[d, bend right] \\ \mathbb{Z}_{\mathrm{sgn}} \arrow[u, bend right] \arrow[loop below] \end{tikzcd}$. Show that this is necessarily a "split" Mackey functor, in the sense that both the restriction and transfer maps are necessarily zero.

2. Recall that the Burnside ring for $G = C_2$ is $A(C_2) \cong \mathbb{Z}\{1, C_2\}$ and that the Burnside Mackey functor $\underline{A}_{C_2}$ is as given to the right.

$$\begin{tikzcd} \mathbb{Z}\{1, C_2\} \arrow[d, bend right, swap, "\begin{pmatrix} 1 & 2 \end{pmatrix}"] \\ \mathbb{Z} \arrow[u, bend right, swap, "\begin{pmatrix} 0 \\ 1 \end{pmatrix}"] \end{tikzcd}$$

   (a) Determine the *ring* structure on $A(C_2)$.

   (b) Show that the linearization homomorphism $\underline{A}_{C_2} \longrightarrow \underline{RO}_{C_2}$ is an isomorphism (this is special to the case $G = C_2$).

   (c) Determine the kernel and cokernel of the map of Mackey functors $\underline{A}_{C_2} \longrightarrow Q(\mathbb{Z}[C_2])$ described in Example 1.71.

   (d) Determine the (unique) map of Mackey functors $\underline{A}_{C_2} \longrightarrow \underline{\mathbb{Z}}$ that is the identity at the trivial subgroup.

   (e) Show that there is *no* map of Mackey functors $\underline{A}_{C_2} \longrightarrow \underline{\mathbb{Z}}^*$ that is the identity at the trivial subgroup. Describe the map that is given as multiplication by 2 at the trivial subgroup.