Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 8

1. Use the comparison of resolutions in Example 1.3.23 to show directly that x_1^2 is nonzero in $\mathrm{H}^2(C_2;\mathbb{F}_2)$.

2. In this example you will describe the cohomology ring of $\mathrm{H}^*(C_3;\mathbb{F}_3)$.

   (a) As in Example 1.3.25, use the long exact sequence in cohomology from the short exact sequence $\mathbb{Z} \xrightarrow{3} \mathbb{Z} \longrightarrow \mathbb{F}_3$ of coefficients to find the cohomology groups $\mathrm{H}^*(C_3;\mathbb{F}_3)$.

   (b) Let y_2 be a nontrivial element in $\mathrm{H}^2(C_3;\mathbb{F}_3)$. Show that the powers of y_2 are nonzero. (Hint: follow the reasoning in Example 1.3.25.)

   (c) Let x_1 be a nontrivial element in $\mathrm{H}^1(C_3;\mathbb{F}_3)$. Use graded-commutativity to show that $x_1^2 = 0$.

   The rest of the argument given in Example 1.3.25 shows that every class in odd degree is of the form $\pm y_2^n \cdot x_1$, and we conclude that

$$\mathrm{H}^*(C_3;\mathbb{F}_3) \cong \mathbb{F}_3[x_1, y_2]/(x_1^2).$$

3. (a) Writing $P_*^{C_k}$ for the free resolution of $\mathbb{Z}$ over C_k, build a comparison of resolutions $P_*^{C_3} \longrightarrow \downarrow_{C_3}^{C_6} P_*^{C_6}$ in degrees 0, 1, and 2.

   (b) Use your comparison in (a) to describe the map $\mathrm{H}^2(C_6) \longrightarrow \mathrm{H}^2(C_3)$ and the resulting ring homomorphism $\mathrm{H}^*(C_6) \longrightarrow \mathrm{H}^*(C_3)$.

   (c) Writing $p\colon C_6 \longrightarrow C_2$ for the quotient, build a comparison of resolutions $P_*^{C_6} \longrightarrow p^* P_*^{C_2}$ in degrees 0, 1, and 2.

   (d) Use your comparison in (c) to describe the map $\mathrm{H}^2(C_2) \longrightarrow \mathrm{H}^2(C_6)$ and the resulting ring homomorphism $\mathrm{H}^*(C_2) \longrightarrow \mathrm{H}^*(C_6)$.