# Math 751 – Fall 2020
# Equivariant homotopy and cohomology
# Worksheet 12

1. Let $H$ and $K$ be subgroups of $G$.

   (a) Let $\gamma K \in G/K$. Show that the stabilizer of $\gamma K$ is $K^\gamma = \gamma K \gamma^{-1}$. (Recall that the stabilizer of $x \in X$ is the set of $g \in G$ which fix $x$.) In other words, show that $g \cdot \gamma K = \gamma K$ in $G/K$ if and only if $g \in K^\gamma$.

   (b) Let $\varphi\colon G/H \longrightarrow G/K$ be defined by $\varphi(gH) = g \cdot \gamma K$. Show that $\varphi$ is well-defined if and only if $H$ is contained in $K^\gamma = \gamma K \gamma^{-1}$.

   (c) Conclude that $\mathrm{Aut}_{\mathcal{O}_G}(G/H) \cong N_G(H)/H = W_G(H)$.

2. (a) Prove Lemma 2.1.60: Let $C$ be a $G$-coefficient system. Then

$$\mathrm{Hom}_{\mathrm{Coeff}}(C, \underline{\mathbb{Z}}) \cong \mathrm{Hom}_{\mathbf{AbGp}}(C(e)/G, \mathbb{Z}).$$

   (b) Prove Lemma 2.1.66: Let $C$ be a $G$-coefficient system. Then

$$\mathrm{Hom}_{\mathrm{Coeff}}(C, \mathrm{Inf}_{G/G}^{G}(\mathbb{Z})) \cong \mathrm{Hom}_{\mathbf{AbGp}}(C(G), \mathbb{Z}).$$

   (c) Prove Lemma 2.1.68: Let $C$ be a $G$-coefficient system. Then

$$\mathrm{Hom}_{\mathrm{Coeff}}(C, \uparrow_e^G (\mathbb{Z})) \cong \mathrm{Hom}_{\mathbf{AbGp}}(C(e), \mathbb{Z}).$$