Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 10

1. Recall that we defined coinduction for H-modules by the formula

$$\mathrm{Coind}_H^G(V) = \mathrm{Hom}_{\mathbf{k}[H]}(\mathbf{k}[G], V).$$

We can make an analogous definition for an H-equivariant space:

$$\mathrm{Coind}_H^G(X) = \mathrm{Map}_H(G, X),$$

where $\mathrm{Map}_H(-,-)$ denotes the space of H-equivariant maps.

   (a) Show that coinduction is right adjoint to the restriction $\downarrow_H^G \colon G\mathbf{Top} \longrightarrow H\mathbf{Top}$ (like Proposition 1.1.45).

   (b) Let X be a finite set of cardinality k and denote by n the order of G. What is the cardinality of the induced G-set $G \times X$? What is the cardinality of the coinduced G-set $\mathrm{Coind}_e^G(X)$?

   Conclude that, in *contrast* to the algebraic result Proposition 1.1.46, induction and coinduction do *not* agree in the topological (or discrete) setting.

   (c) If X is based, then what is a natural choice of basepoint for $\mathrm{Coind}_H^G(X)$? In this situation, find a natural isomorphism

$$\mathrm{Coind}_H^G(X) \cong \mathrm{Map}_{*,H}(G_+, X),$$

   where $\mathrm{Map}_{*,H}(-,-)$ denotes the space of *based* H-equivariant maps and G_+ denotes G equipped with a disjoint basepoint.

2. In this problem, you will find some K_4-equivariant CW structures on representation spheres.

   (a) Find an equivariant cell structure on the K_4-space $S^{p_1^*(\sigma)}$.

   (b) How would the discussion in part (a) change to give a cell structure on $S^{m^*(\sigma)}$ or $S^{p_2^*(\sigma)}$?

   (c) Find an equivariant cell structure on the K_4-space $S^{p_1^*(\sigma)} \wedge S^{p_2^*(\sigma)} \cong S^{p_1^*(\sigma) \oplus p_2^*(\sigma)}$.

   There are several ways to think about this problem. Use whichever you prefer.

      i. One way is to follow the same approach that is used for obtaining cell structures on (smash) products nonequivariantly.

      ii. Another approach is to first find a K/L-equivariant cell structure on the space of L-fixed points, and similarly for the D- and R-fixed points. Any point that is not fixed by one of these subgroups must be contained in a free cell.