Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 1

1. Let S be a (possibly noncommutative) ring, and let M be a left S-module. Suppose that $\varphi\colon M \longrightarrow M$ is an S-module map that is idempotent, meaning that $\varphi \circ \varphi = \varphi$.

   (a) Show that $1 - \varphi$ is also an idempotent S-module map.

   (b) Show that M is isomorphic to $\mathrm{im}(\varphi) \oplus \ker(\varphi)$.

2. Consider $\mathbb{C}$ as a two-dimensional real representation of C_2, where τ acts as complex conjugation. Show that this representation is isomorphic to the regular representation of C_2.

3. For the groups $G = C_n$, $C_p \times C_q$, and D_n, write the group rings over a ring R as a quotient of a (possibly noncommutative) polynomial algebra.

4. Let G and H be groups.

   (a) Show that an action of the group $G \times H$ on a set X amounts to a pair of commuting actions of G and H on X. Commuting here means that $g \cdot (h \cdot x) = h \cdot (g \cdot x)$.

   (b) In the Dihedral Representations video, I described the four irreducible one-dimensional representations of $C_2 \times C_2$: the trivial representation and the three sign representations. Describe these representations from the "pair of commuting actions" point of view.