Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 3

1. Recall that $RO(K_4) \cong \mathbb{Z}\{\mathbf{1}, p_1^*(\sigma), m^*(\sigma), p_2^*(\sigma)\}$. Determine the restriction homomorphisms $RO(K_4) \longrightarrow RO(C_2)$, for the three subgroups L (left), D (diagonal), and R (right) of order two.

2. In this problem, you will consider restriction and induction homomorphisms between the representation rings for the subgroups of D_3. We write C_3 for the subgroup generated by r and C_2 for the subgroup generated by s.

   (a) Find formulas for the restriction and induction homomorphisms
   $$\mathbb{Z} = RO(e) \rightleftarrows RO(C_2), \qquad \mathbb{Z} = RO(e) \rightleftarrows RO(C_3).$$

   (b) Find formulas for the restriction homomorphisms
   $$\downarrow_{C_2}^{D_3} \colon RO(D_3) \longrightarrow RO(C_2), \qquad \downarrow_{C_3}^{D_3} \colon RO(D_3) \longrightarrow RO(C_3).$$

   (c) Use the double coset formula to show that the composition
   $$RO(C_3) \xrightarrow{\uparrow} RO(D_3) \xrightarrow{\downarrow} RO(C_3)$$
   is multiplication by 2. Use this to conclude that
   $$\uparrow_{C_3}^{D_3} (\lambda_3) = 2\nu_1 \quad \text{and} \quad \uparrow_{C_3}^{D_3} (\mathbf{1}) = \mathbf{1} \oplus \sigma.$$

   (d) Show that there is only one double coset $C_3 \backslash D_3 / C_2$, and use the double coset formula to give a formula for the composition
   $$RO(C_2) \xrightarrow{\uparrow} RO(D_3) \xrightarrow{\downarrow} RO(C_3).$$
   Use this to deduce the values of
   $$\uparrow_{C_2}^{D_3} \colon RO(C_2) \longrightarrow RO(D_3).$$

3. This problem concerns the Klein four-group K_4.

   (a) Show that $\uparrow_L^K (\mathbf{1}) = \mathbf{1} \oplus p_2^*(\sigma)$. (This can be shown directly).

   (b) Use the projection formula to deduce that
   $$\uparrow_L^K (\sigma) = p_1^*(\sigma) \oplus \big(p_1^*(\sigma) \otimes p_2^*(\sigma)\big),$$
   and show that $p_1^*(\sigma) \otimes p_2^*(\sigma)$ is $m^*(\sigma)$.