Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 9

1. In this problem, you will determine the cohomology ring $\mathrm{H}^*(C_4; \mathbb{F}_2)$.

   (a) Use the long exact sequence in cohomology arising from the short exact sequence $\mathbb{Z} \xrightarrow{2} \twoheadrightarrow \mathbb{F}_2$ to compute the groups $\mathrm{H}^*(C_4; \mathbb{F}_2)$.

   (b) Use the results from Example 1.3.26 to deduce the map $i^*\colon \mathrm{H}^*(C_4; \mathbb{F}_2) \longrightarrow \mathrm{H}^*(C_2; \mathbb{F}_2)$, where $i\colon C_2 \hookrightarrow C_4$ is the inclusion.

   (c) Use your results above to deduce the ring structure on $\mathrm{H}^*(C_4; \mathbb{F}_2)$.

2. In this problem, you will consider the **shearing isomorphism**. Let X be a G-space.

   (a) Consider the product G-space $G \times X$, where the group action is on *both* factors. We will also write X_{triv} for the same space, but equipped with a trivial action. Find an equivariant isomorphism
   $$G \times X_{\mathrm{triv}} \cong G \times X.$$

   (b) Generalizing the previous problem, for any subgroup $H \leq G$, find a G-equivariant isomorphism
   $$G \times_H \downarrow_H^G X \cong G/H \times X,$$
   where G acts on $G \times_H \downarrow_H^G X$ as left multiplication on G.

   (c) Part (b) can be summarized by the formula
   $$\uparrow_H^G \downarrow_H^G (X) \cong G/H \times X.$$
   This formula works equally well when X is a finite G-set. Explore this formula in the case of $G = C_6$, $H = C_3$, and X is a C_6-orbit.