Math 751 – Fall 2020
Equivariant homotopy and cohomology
Worksheet 6

1. Recall that an inflated Mackey functor $\mathrm{Inf}_e^{C_2} B$ for the group C_2 has value B at C_2 (the top) and zero at the bottom.

   (a) Show that the only maps (in either direction) between the C_2-Mackey functors $\underline{\mathbb{Z}}$ and $\mathrm{Inf}_e^{C_2}\mathbb{Z}$ are the zero maps.

   (b) Describe a surjection $\underline{\mathbb{Z}} \twoheadrightarrow \mathrm{Inf}_e^{C_2}\mathbb{F}_2$, and describe the kernel.

2. For any group G, the first step in a free resolution of the $\mathbb{Z}[G]$-module $\mathbb{Z}$ is the surjection $\varepsilon\colon \mathbb{Z}[G] \twoheadrightarrow \mathbb{Z}$ sending each generator g to 1.

   (a) The kernel of $\varepsilon\colon \mathbb{Z}[G] \twoheadrightarrow \mathbb{Z}$ is known as the **augmentation ideal** and written $I(G) \subset \mathbb{Z}[G]$. Show that $I(G)$ is free abelian and generated by the differences $g - e$, where g runs over the non-identity elements of G.

   (b) Show that if $G \cong C_n$ is cyclic of order n with generator γ, then the map

   $$\delta\colon \mathbb{Z}[G] \longrightarrow I(G)$$

   defined by $\delta(1) = \gamma - e$ is surjective, with kernel $\mathbb{Z}\{e + \gamma + \cdots + \gamma^{n-1}\}$.

   (c) In the case of $G = C_2 \times C_2$, show that $I(C_2 \times C_2)$ is *not* a cyclic $\mathbb{Z}[C_2 \times C_2]$-module, meaning that there is *no* surjection

   $$\mathbb{Z}[C_2 \times C_2] \longrightarrow I(C_2 \times C_2).$$