Problem 1. (i) Let \( p : E \rightarrow X \) be the universal cover. Show that, for \( n \geq 2 \), the action of \( \pi_1 (X) \) on \( \pi_n (X) \cong \pi_n (E) \) corresponds to the action of deck transformations on \( E \).
(ii) Show that \( \pi_1 (\mathbb{R}{\mathbb{P}}^n) \) acts trivially on \( \pi_n (\mathbb{R}{\mathbb{P}}^n) \) if and only if \( n \) is odd.

Problem 2. Let \( F \xrightarrow{i} E \xrightarrow{p} B \) be a fiber sequence of connected spaces.
(i) Construct an action of \( \pi_1 (E) \) on the higher homotopy groups of the fiber \( F \). Hint: given \( \alpha \in \pi_n (F) \) and \( \gamma \in \pi_1 (E) \), consider \( p_* (\gamma \cdot i_* (\alpha)) \in \pi_n (B) \).
(ii) Find an example in which \( E \) is simple, but \( F \) is not. (You should be able to find an example in which \( \pi_1 (F) \) is not abelian.)

Problem 3. Let \( X \) be a based space, let \( n \geq 0 \), and let \( A \) be any abelian group. Show that there is a weak equivalence
\[
\text{Map}_* (X, K(A, n)) \simeq \prod_{0 \leq i \leq n} K(\tilde{H}^i (X; A), n - i).
\]

Problem 4. Determine the second Postnikov section \( P_2 (\Omega S^2) \). Show that each \( P_n (\Omega S^2) \) is equivalent to a product \( S^1 \times Y_n \), and describe the space \( Y_n \). Show, however, that \( \Omega S^2 \) is not a Generalized Eilenberg-Mac Lane space (GEM).