Problem 1. Show that an injective covering map must be a homeomorphism.

Problem 2. Show that the map \( q : (0, 2) \to S^1 \) defined by \( q(x) = e^{2\pi ix} \) is not a covering map.

Problem 3. Suppose that \( q : E \to B \) is a covering and \( b_1, b_2 \in B \). (As usual, assume that \( E \) is connected and locally path-connected.) Show that there is a bijection \( q^{-1}(b_1) \cong q^{-1}(b_2) \).

Problem 4. A subspace \( A \subseteq X \) is said to be a deformation retract of \( X \) if there is a retraction \( r : X \to A \) and a homotopy \( h : i \circ r \simeq id_X \) such that \( h(a, t) = a \) for all \( a \in A \).

(1) Show the torus with a point removed deformation retracts onto \( \infty \) (= \( S^1 \vee S^1 \)). (Hint: Think of the torus as a quotient of \( I^2 \).)

(2) Show that \( \mathbb{R}^2 \) with two points removed deformation retracts onto \( \infty \).

(3) What do parts (1) and (2) tell you about the fundamental groups of the punctured torus and the twice punctured plane?

Problem 5.

(1) Describe a covering of \( S^1 \vee S^1 \) by the space \( E \) given in the picture below:

(2) Take the point labelled as 0 as the basepoint for \( E \). What is the image under your covering map \( p \) of the loop around the circle at 0? What about the loop (at 0) around the circle at 1?

(3) Show that the two loops in \( E \) described in part (2) are not homotopic. Use this to show that \( \pi_1(S^1 \vee S^1) \) is not abelian.

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