MATH 651
HOMEWORK VII
SPRING 2013

Problem 1. Show that if $g : A \longrightarrow Y$ is surjective, then so is $\iota_X : X \longrightarrow X \cup_A Y$.

Problem 2. Let $A \subseteq X$ be a subspace, and let $f : A \longrightarrow X$ be the inclusion. As usual, we let $*$ denote a one-point space. Show that $X \cup_A * \cong X/A$. (**Hint:** Show they satisfy the same universal property.)

Problem 3. For any space A, let $C(A)$ be the cone on A. We can think of A as a subspace of $C(A)$ via the inclusion $i_0 : A \longrightarrow C(A)$ at time 0.

(a) Show that a map $g : A \longrightarrow Y$ is null if and only if it extends to a map $G : C(A) \longrightarrow Y$.

(b) Suppose given a map $f : A \longrightarrow X$ and let $C(f) = X \cup_A C(A)$ be the mapping cone on f. Given a map $\varphi : X \longrightarrow Y$, show that $\varphi \circ f$ is null if and only if φ extends over the mapping cone $C(f)$.

Problem 4.

(1) Let x and y be any two (distinct) points in $\mathbb{R}^3$. Use the van Kampen theorem to compute $\pi_1(\mathbb{R}^3 - \{x, y\})$.

(2) If a third point z is thrown into the mix, what is the resulting fundamental group (of $\mathbb{R}^3 - \{x, y, z\}$)?

Problem 5. Let X be $\mathbb{R}^3$ with two of the coordinate axes removed. Compute $\pi_1(X)$. (**Hint:** Start by showing that X is homotopy equivalent to S^2 with four points removed.)

Date: March 9, 2013.