Problem 1. Show that if \( g : A \to Y \) is surjective, then so is \( \iota_X : X \to X \cup_A Y \).

Problem 2. Let \( A \subseteq X \) be a subspace, and let \( f : A \to X \) be the inclusion. As usual, we let \(*\) denote a one-point space. Show that \( X \cup_A * \cong X/A \). (Hint: Show they satisfy the same universal property.)

Problem 3. For any space \( A \), let \( C(A) \) be the cone on \( A \). We can think of \( A \) as a subspace of \( C(A) \) via the inclusion \( i_0 : A \to C(A) \) at time 0.

(a) Show that a map \( g : A \to Y \) is null if and only if it extends to a map \( G : C(A) \to Y \).

(b) Suppose given a map \( f : A \to X \) and let \( C(f) = X \cup_A C(A) \) be the mapping cone on \( f \).
Given a map \( \varphi : X \to Y \), show that \( \varphi \circ f \) is null if and only if \( \varphi \) extends over the mapping cone \( C(f) \).

Problem 4.

(1) Let \( x \) and \( y \) be any two (distinct) points in \( \mathbb{R}^3 \). Use the van Kampen theorem to compute \( \pi_1(\mathbb{R}^3 - \{x, y\}) \).

(2) If a third point \( z \) is thrown into the mix, what is the resulting fundamental group (of \( \mathbb{R}^3 - \{x, y, z\} \))? 

Problem 5. Let \( X \) be \( \mathbb{R}^3 \) with two of the coordinate axes removed. Compute \( \pi_1(X) \). (Hint: Start by showing that \( X \) is homotopy equivalent to \( S^2 \) with four points removed.)