Problem 1. Compute the Euler characteristic of a Möbius band.

Problem 2. Suppose that two finite CW complexes $X_1$ and $X_2$ are homeomorphic. In other words, we have two cell complex structures on the same space, each having finitely many cells. Suppose furthermore that $X_{12}$ is a common finite refinement of the two. In other words, every $n$-cell of $X_1$ is a union of $n$-cells of $X_{12}$, and similarly every $k$-cell of $X_2$ is a union of $k$-cells of $X_{12}$. (Here an $n$-cell means the image of $D^n$ in the pushout.)

(a) If $X_1$ and $X_2$ are both graphs (i.e. 1-dimensional), show that $\chi(X_1) = \chi(X_2)$.
(b) Show that this formula still holds when the dimension (of both) is $n$ for any $n \geq 1$.

Problem 3. Let $X$ be the quotient of $S^2$ obtained by identifying the north and south poles to a single point. Put a cell complex structure on $X$ and use this to compute $\pi_1(X)$.

Problem 4. Let $X$ and $Y$ be finite CW complexes.

(a) Use the cell structures on $X$ and $Y$ to put a CW structure on $X \times Y$. (Hint: It may help to show that $S^{m+n-1} \cong (S^{m-1} \times D^n) \cup_{S^{m-1} \times S^{n-1}} (D^m \times (S^{n-1}))$.)
(b) Use this to deduce a formula for $\chi(X \times Y)$ in terms of $\chi(X)$ and $\chi(Y)$.

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