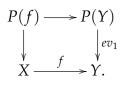
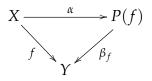
## Math 751 - Topics in Topology Homework 4 Spring 2015

1. (a) (Replacing a map by a fibration) Let  $f : X \longrightarrow Y$  be a map. Let P(Y) be the path space on Y. This is the space of paths in Y, topologized as P(Y) = Map(I, Y). Now define the mapping path space P(f) by the pullback



Show that we have a commutative triangle



in which  $\alpha$  is a homotopy equivalence and  $\beta_f$  is a fibration.

- (b) What do you get if you replace  $* \longrightarrow Y$  by a fibration?
- 2. Suppose that  $f : X \longrightarrow Y$  is a fibration and Y is path-connected. Show that if y and y' are points of Y, then  $f^{-1}(y) \simeq f^{-1}(y')$ . This means that it still makes sense to talk about "the" fiber of a fibration that is not necessarily a bundle.
- 3. Let  $p : E \longrightarrow B$  be a fibration and let  $\iota : A \longrightarrow B$  be the inclusion of a subspace.
  - (a) Show that the restriction  $p: p^{-1}(A) \longrightarrow A$  is also a fibration.
  - (b) Show that if  $\iota : A \longrightarrow B$  induces an isomorphism in homotopy groups  $\pi_n$  for n < k and a surjection on  $\pi_k$ , then the same is true of the inclusion  $j : p^{-1}(A) \longrightarrow E$ . (Such a map is called an *n*-equivalence.)