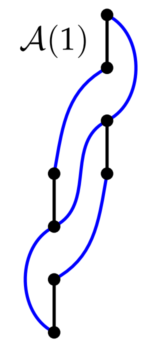
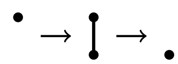
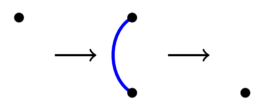
Math 752 - Hopf algebras
Worksheet 6
Spring 2017



1. (From Worksheet 5) Recall that $\mathcal{A}(1) \subseteq \mathcal{A}$ is the subalgebra generated by $\boxed{1}$ and $\boxed{2}$. The algebra generators $\boxed{1}$ and $\boxed{2}$ give rise to elements $h_0 \in \mathrm{Ext}^{1,1}_{\mathcal{A}(1)}$ and $h_1 \in \mathrm{Ext}^{1,2}_{\mathcal{A}(1)}$ which can be thought of as the extensions.

h_0 :  and h_1 : 

   (a) Show that $h_0 h_1 = 0$ in $\mathrm{Ext}^{2,3}_{\mathcal{A}(1)}$ using the extension approach.

   (b) Show that $h_0 h_1 = 0$ in $\mathrm{Ext}^{2,3}_{\mathcal{A}(1)}$ from the cobar complex. Recall that $\mathcal{A}(1)^\vee \cong \mathbf{F}_2[z_1, z_2]/(z_1^4, z_2^2)$, with z_1 primitive and $\Delta(z_2) = z_1^2 \otimes z_1$.

   (c) Show (using either method) that $h_1^3 = 0$.

2. Recall that in Wednesday's class, we started to investigate the Cartan-Eilenberg spectral sequence for the central extension

$$E(Q_1) \longrightarrow \mathcal{A}(1) \longrightarrow \mathcal{A}(1)/Q_1,$$

with $\mathrm{H}^*(E(Q_1)) \cong \mathbf{F}_2[v_1]$ and $\mathrm{H}^*(\mathcal{A}(1)/Q_1) \cong \mathbf{F}_2[h_0, h_1]$.

   (a) Last time, we established the differential $d_2(v_1) = h_0 h_1$. Deduce that $E_3 \cong \mathbf{F}_2[v_1^2, h_0, h_1]/h_0 h_1$.

   (b) Either by considering the cobar complex or by building a minimal resolution, show that v_1^2 cannot survive to $\mathrm{H}^2(\mathcal{A}(1))$. Use this to deduce the differential $d_3(v_1^2) = h_1^3$.

   (c) Deduce from (b) that E_4 has a vector space basis

$$E_4 \cong \mathbf{F}_2\{h_0^n \cdot v_1^{4j}, h_1^k \cdot v_1^{4j}, h_0^n v_1^{4j} \cdot h_0 v_1^2\}_{n,j \geq 0,\ 2 \geq k \geq 0}$$

   and show that $E_4 = E_\infty$ by showing that v_1^4 cannot support any differentials.