Math 752 - Hopf algebras
Worksheet 7
Spring 2017

1. Flesh out the May spectral sequence computation of $\mathrm{Ext}_{\mathcal{A}}$ indicated in class for $t-s \leq 13$. Here is some extra information about the May spectral sequence: for the purpose of this spectral sequence, we declare that $h_{i,j}$ lives in May filtration i. The May differential d_r decreases the May filtration by $r-1$. It also decreases $t-s$ by 1 and increases s by 1. (Up 1 and left 1 in the attached picture).

   As you work through items (a)-(e) below, you should be generating a picture for the E_4-page.

   (a) Use the Adams vanishing theorem to deduce the differential $d_2(b_{20}) = h_0^2 h_2 + h_1^3$.

   (b) Use the relations in E_2 to deduce $d_2(h_0(1)) = h_0 h_2^2$.

   (c) Use Adams vanishing and the E_2-relations to deduce $d_2(b_{21}) = h_2^3 + \delta h_1^2 h_3$, where $\delta \in \{0, 1\}$.

   (d) Use Adams vanishing to deduce that $d_2(b_{30}) = h_3 b_{20} + \varepsilon \cdot h_1 b_{21}$, where $\varepsilon \in \{0, 1\}$.

   (e) Use the fact that $d_2^2 = 0$ to deduce that $\delta = 1$ in part (c) and $\varepsilon = 1$ in part (d).

   (f) Use Adams vanishing to deduce that $d_4(b_{20}^2) = h_0^4 h_3$.

   (g) Proudly display your picture of $E_6 = E_\infty$ and compare to the picture of $\mathrm{Ext}_{\mathcal{A}}$.

2. (The doubling isomorphism)

   (a) Let $\mathcal{AA}_* \subseteq \mathcal{A}_*$ be the subalgebra generated by $(z_1^2, z_2^2, \dots)$. Convince yourself that the degree-halving map $h : \mathcal{AA}_* \longrightarrow \mathcal{A}_*$ defined on generators by $h(z_i^2) = z_i$ is an isomorphism of Hopf algebras.

   (b) Deduce that there is a dual, degree doubling isomorphism $D : \mathcal{A} \longrightarrow \mathcal{A}//\mathcal{E}$.

   (c) Show that this restricts to an isomorphism $D : \mathcal{A}(n-1) \longrightarrow \mathcal{A}(n)//\mathcal{E}(n)$.