1. Flesh out the May spectral sequence computation of $\operatorname{Ext}_A$ indicated in class for $t - s \leq 13$. Here is some extra information about the May spectral sequence: for the purpose of this spectral sequence, we declare that $h_{i,j}$ lives in May filtration $i$. The May differential $d_r$ decreases the May filtration by $r - 1$. It also decreases $t - s$ by 1 and increases $s$ by 1. (Up 1 and left 1 in the attached picture).

As you work through items (a)-(e) below, you should be generating a picture for the $E_4$-page.

(a) Use the Adams vanishing theorem to deduce the differential $d_2(b_{20}) = h_0^2h_2 + h_1^3$.
(b) Use the relations in $E_2$ to deduce $d_2(h_0(1)) = h_0h_2^2$.
(c) Use Adams vanishing and the $E_2$-relations to deduce $d_2(b_{21}) = h_2^3 + \delta h_1^2h_3$, where $\delta \in \{0,1\}$.
(d) Use Adams vanishing to deduce that $d_2(b_{30}) = h_5b_{20} + \epsilon \cdot h_1b_{21}$, where $\epsilon \in \{0,1\}$.
(e) Use the fact that $d_2^2 = 0$ to deduce that $\delta = 1$ in part (c) and $\epsilon = 1$ in part (d).
(f) Use Adams vanishing to deduce that $d_4(b_{20}^2) = h_0^4h_3$.
(g) Proudly display your picture of $E_6 = E_\infty$ and compare to the picture of $\operatorname{Ext}_A$.

2. (The doubling isomorphism)

(a) Let $AA_* \subseteq A_*$ be the subalgebra generated by $(z_1^2, z_2^2, \ldots)$. Convince yourself that the degree-halving map $h : AA_* \rightarrow A_*$ defined on generators by $h(z_i^2) = z_i$ is an isomorphism of Hopf algebras.
(b) Deduce that there is a dual, degree doubling isomorphism $D : A \rightarrow A//E$.
(c) Show that this restricts to an isomorphism $D : A(n-1) \rightarrow A(n)//E(n)$. 