1. Recall from class that if $f_*$ and $g_*$ are chain maps $C_* \to D_*$, then a chain-homotopy between $f_*$ and $g_*$ is a sequence of homomorphisms $h_n : C_n \to D_{n+1}$ such that 
\[ \partial_{n+1}^D(h_n(c)) + h_{n-1}(\partial_n^C c) = g(c) - f(c). \]
(a) Given a chain complex $C_*$, define a new chain complex $C_* \otimes I_*$ as follows: 
\[ (C_* \otimes I_*)_n := C_n\{v_0\} \oplus C_n\{v_1\} \oplus C_{n-1}\{e\}, \]
and 
\[ \partial \begin{pmatrix} c_0 \\ c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \partial(c_0) + (-1)^n c_2 \\ \partial(c_1) + (-1)^{n+1} c_2 \\ \partial(c_2) \end{pmatrix}. \]
Check that this really defines a chain complex, meaning that $\partial^2 = 0$.
(b) Define two chain maps $C_* \to C_* \otimes I_*$, playing the role of $i_0$ and $i_1$.
(c) Check that a chain map $h : C_* \otimes I_* \to D_*$ corresponds to a chain homotopy between $h \circ i_0$ and $h \circ i_1$.
(d) Show that if $f_*$ and $g_*$ are chain-homotopic, then $f_*$ and $g_*$ induce the same map on homology.

2. Find an example of a chain map that induces an isomorphism on homology but is not a chain-homotopy equivalence.

3. A short exact sequence is a sequence of homomorphisms 
\[ 0 \to A \overset{i}{\to} B \overset{p}{\to} C \to 0 \]
which is exact (has trivial homology) at each spot. A short exact sequence is called split exact if $B \cong A \oplus C$. Show that the following are equivalent for the (solid arrow) sequence:
\[ 0 \to A \overset{i}{\to} B \overset{p}{\to} C \to 0 \]
(a) The sequence is split exact
(b) There exists a homomorphism $s$ such that $p \circ s = \text{id}_C$ ($s$ is called a splitting)
(c) There exists a homomorphism $r$ such that $r \circ i = \text{id}_A$ ($r$ is called a retraction or splitting)