Math 751 – Spring 2024
Chromatic Homotopy Theory
Worksheet 2

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1. An **additive** category is a category enriched in abelian groups (meaning that each Hom set has an abelian group structure such that composition is bilinear) and which has all finite products.

   Show if A and B are objects of an additive category $\mathcal{C}$, then $A \times B$ also serves as a coproduct of A and B. It is then common to write $A \oplus B$ for this product/coproduct.

In the following problem, you may use the following facts about a sequence of maps $X_0 \to X_1 \to X_2 \to \dots$:

(a) Compactness of S^1 implies that there is a canonical weak homotopy equivalence
$$\operatorname{hocolim}_k \Omega X_k \xrightarrow{\sim} \Omega \operatorname{hocolim}_k X_k.$$

(b) Passing to homotopy classes induces an isomorphism
$$\pi_n \operatorname{hocolim}_k X_k \cong \operatorname{colim}_k \pi_n X_k.$$

2. Let E be a spectrum.

   (a) For each $n \geq 0$, define a based space RE_n by $RE_n = \operatorname{hocolim}_k \Omega^k E_{n+k}$. Use these to define a spectrum RE.

   (b) Write down a stable equivalence $E \xrightarrow{\sim} RE$.

3. Show that $D\mathbb{S}/n$, the dual of $\mathbb{S}/n$, is $\mathbb{S}^{-1}/n$.

   (Hint: the functor $F(-, \mathbb{S})$ takes cofiber sequences to fiber sequences.)