Math 751 – Spring 2024
Chromatic Homotopy Theory
Worksheet 11

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1. (a) Show that $K(n)_* H\mathbb{F}_p \cong 0$ for all n.
   (Hint: $H\mathbb{F}_p$ can be realized as the regular quotient $BP/(v_0, v_1, v_2, \dots)$. Show that $K(n)_*(BP/(v_0, \dots, v_n))$ is already zero.)
   (b) Conclude that $\widehat{L_n} H\mathbb{F}_p \simeq *$ for all n.
   (c) Conclude that $L_n H\mathbb{F}_p \simeq *$ for all n.
   (d) Conclude that the chromatic tower for $H\mathbb{F}_p$ fails to converge.

2. Define $M_n X$ to be the fiber of $L_n X \to L_{n-1} X$. This is called the nth **monochromatic layer** of X. (We define $L_{-1} X = *$, so that $M_0 X = L_0 X$.) In this problem, feel free to use (or prove) that M_n preserves fiber sequences.
   (a) Show that $M_n L_{n-1} X \simeq *$.
   (b) Show that $M_n X \to M_n L_{K(n)} X$ is an equivalence.
   (c) Show that $K(n) \wedge L_{n-1} X \simeq *$.
   (d) Show that $L_{K(n)} M_n X \to L_{K(n)} L_n X \simeq L_{K(n)}$ is an equivalence.
   (e) Deduce that M_n and $L_{K(n)}$ induce equivalences of categories between the homotopy category of $K(n)$-local spectra and the n-th monochromatic category, defined to be the image of M_n on Ho**Sp**.