Math 751 – Spring 2024
Chromatic Homotopy Theory
Worksheet 10

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1. Prove Proposition 8.11: If R is a commutative hring spectrum and $(x_1, x_2, \dots)$ is a regular sequence in R_*, then

$$\pi_* \left(R/(x_1, x_2, \dots)\right) \cong R_*/(x_1, x_2, \dots).$$

   Feel free to use that, by definition, $R/(x_1, x_2, \dots)$ is the homotopy colimit over n of $R/(x_1, \dots, x_n)$.

2. Let $x \in \pi_n \mathbb{S}$. A **left unital pairing** on $\mathbb{S}/x$ is a map $\mu \colon \mathbb{S}/x \wedge \mathbb{S}/x \to \mathbb{S}/x$ such that the composition

$$\mathbb{S}/x = \mathbb{S} \wedge \mathbb{S}/x \xrightarrow{q \wedge \mathrm{id}} \mathbb{S}/x \wedge \mathbb{S}/x \xrightarrow{\mu} \mathbb{S}/x$$

   is the identity. Show that a left unital pairing exists on $\mathbb{S}/x$ if and only if the identity map on $\mathbb{S}/x$ is annihilated by x. (Hint: smash the cofiber sequence $\mathbb{S}^n \xrightarrow{x} \mathbb{S} \longrightarrow \mathbb{S}/x$ with $\mathbb{S}/x$.)

   Note that since $\pi_2(\mathbb{S}/2)$ is $\mathbb{Z}/4$, this shows that $\mathbb{S}/2$ cannot even have a left unital pairing.

3. Show that the truncated Brown-Peterson spectrum $BP\langle n \rangle$ is not Landweber exact.

4. Recall that we showed that the Moore spectrum $\mathbb{S}/3$ admits a v_1-self-map $\Sigma^4 \mathbb{S}/3 \xrightarrow{v_1} \mathbb{S}/3$.

   (a) Find $k(1)_* \mathbb{S}/3$ and $K(1)_* \mathbb{S}/3$. Note that the spectra $k(1)$ and $K(1)$ depend on a choice of prime p. How does your answer change, depending on the prime?

   (b) Write $\mathbb{S}/(3, v_1)$ for the cofiber of the v_1-self-map on $\mathbb{S}/3$. Compute $k(1)_* \mathbb{S}/(3, v_1)$ and $K(1)_* \mathbb{S}/(3, v_1)$.