

Math 751 – Spring 2024

Chromatic Homotopy Theory

Worksheet 7

1. Verify that the homotopy groups of KO and KU are compatible, in the sense that the Wood cofiber sequence

$$\Sigma^1 KO \xrightarrow{\eta} KO \longrightarrow KU$$

is possible. In other words, check that the cofiber of η on KO has the same homotopy groups as KU .

2. Determine as much of the homotopy groups of the fiber of $KO_2^\wedge \xrightarrow{\psi^3 - \text{id}} KO_2^\wedge$ as you can.
3. Show that if X is $E(1)$ -local, then X/p is $K(1)$ -local. (Hint: use the fracture square.)
4. For $p = 3$, verify that the homotopy groups of $L_1(\mathbb{S}/3)$ are v_1 -periodic. In other words, compute the homotopy groups and observe the v_1 -periodicity.