# Math 751 – Spring 2024
# Chromatic Homotopy Theory
# Worksheet 8

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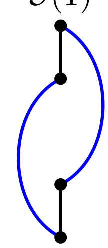
**Strategy for building a resolution:** Say we want to "resolve" $\mathbf{k}$ over some algebra $\mathcal{R}$. Start by choosing a free $\mathcal{R}$-module $P_0$ with a surjection $P_0 \longrightarrow \mathbf{k}$. Write $K_0$ for the kernel of $P_0 \to \mathbf{k}$. Then find a free $\mathcal{R}$-module $P_1$ with a surjection $P_1 \twoheadrightarrow K_0$ and write $K_1$ for the kernel of $P_1 \twoheadrightarrow K_0$. Repeating in this way, one produces a structure of the form

$$\begin{tikzcd}
{} \arrow[r] & P_3 \arrow[rr] \arrow[dr, two heads] & & P_2 \arrow[rr] \arrow[dr, two heads] & & P_1 \arrow[rr] \arrow[dr, two heads] & & P_0 \arrow[r, two heads] & \mathbf{k} \\
& & K_2 \arrow[ur, hook] & & K_1 \arrow[ur, hook] & & K_0 \arrow[ur, hook] & &
\end{tikzcd}$$

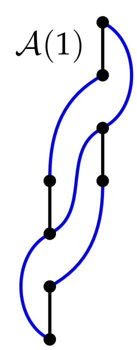
$\mathcal{E}(1)$



1. Build the first 4 stages (i.e., up to $P_3$) of a minimal resolution of $\mathbb{F}_3$ over $\mathcal{E}(1) = E(\beta, Q_1)$, where $Q_1$ is in degree 5. Draw an Adams chart for your resulting partial answer for $\mathrm{Ext}_{\mathcal{E}(1)}\, \mathbb{F}_3, \mathbb{F}_3)$.

   In fact, this is the $E_2$-term for a collapsing Adams spectral sequence that computes the homotopy of the connective cover $\ell_p^\wedge$ of $E(1)_p^\wedge$.

$\mathcal{A}(1)$



2. Build the first 5 stages (i.e., up to $P_4$) of a minimal resolution of $\mathbb{F}_2$ over $\mathcal{A}(1) \subset \mathcal{A}_2$, where $\mathcal{A}(1)$ is displayed to the right. The generators are $\mathrm{Sq}^1 = \beta$ in degree 1 and $\mathrm{Sq}^2 = \mathcal{P}^1$ in degree 2. Draw an Adams chart for your resulting partial answer for $\mathrm{Ext}_{\mathcal{A}(1)}\, \mathbb{F}_2, \mathbb{F}_2)$.

   In fact, this is the $E_2$-term for a collapsing Adams spectral sequence that computes the homotopy of the connective cover $ko_2^\wedge$ of $KO_2^\wedge$.