Worksheet #12: Volumes of Revolution

1. Find the volume of the solid obtained by rotating the region bounded by $y = \dfrac{1}{x^5}$, $y = 0$, $x = 1$, and $x = 6$, about the x-axis.

2. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis. $y = 0$, $y = \cos(2x)$, $x = \frac{\pi}{2}$, $x = 0$ about the line $y = -6$.

3. Find the volume of the solid obtained by rotating the region in the first quadrant bounded by the curves $x = 0$, $y = 1$, $x = y^{11}$, about the line $y = 1$.

4. For each of the following, use disks or washers to find the integral expression for the volume of the region.

   (a) R is the region bounded by $y = 1 - x^2$ and $y = 0$; about the x-axis.

   (b) R is the region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$; about the y-axis.

   (c) R is the region bounded by $y = 1 - x^2$ and $y = 0$; about the line $y = -1$.

   (d) Between the regions in part (a) and part (c), which volume is bigger? Why? First argue without computing the integrals, then also evaluate the integrals to check your answer.

   (e) R is the region bounded by $y = e^{-x}$, $y = 1$ and $x = 2$; about the line $y = 2$.

   (f) R is the region bounded by $y = x$ and $y = \sqrt{x}$; about the line $x = 2$.

5. Find the volume of the cone obtained by rotating the region in the first quadrant under the segment joining $(0, h)$ and $(r, 0)$ about the y-axis.

6. A soda glass has the shape of the surface generated by revolving the graph of $y = 6x^2$ for $0 \leq x \leq 1$ about the y-axis. Soda is extracted from the glass through a straw at the rate of $1/2$ cubic inch per second. How fast is the soda level in the glass dropping when the level is 2 inches? (Answer should be implicitly in units of inches per second.)

7. The torus is the solid obtained by rotating the circle $(x-a)^2 + y^2 = b^2$ around the y-axis (assume that $a > b$). Show that it has volume $2\pi^2 ab^2$.
   [Hint: Draw a picture, set up the problem and evaluate the integral by interpreting it as the area of a circle.]

8. Conceptual understanding of disk and shell method:

   (a) Write a general integral to compute the volume of a solid obtained by rotating the region under $y = f(x)$ over the interval $[a, b]$ about the y-axis using the method of cylindrical shells.

   (b) If you use the disk method to compute the same volume, are you integrating with respect to x or y? Why?

9. Sketch the enclosed region and use the Shell Method to calculate the volume of rotation about the y-axis.

   (a) $y = 3x - 2$, $y = 6 - x$, $x = 0$

   (b) $y = x^2$, $y = 8 - x^2$, $x = 0$, for $x \geq 0$