MA 114 Worksheet # 29: Area and arc length in polar coordinates

1. Given the circle represented by $x^2 + (y-2)^2 = 4$

   (a) Find the polar representation for this equation.

   (b) Calculate the area enclosed by $0 \le \theta \le \pi/4$.

   (c) Sketch the area calculated.

2. The equation $r = 2\sin(2\theta)$ represents the "four petaled rose".

   (a) Find the area of one of the petals of the rose.

   (b) Given the circle $x^2 + y^2 = 1$, find the area between the rose and the circle (using the rose as the outer curve).

   (c) Find the area between the rose and the circle using the circle as the outer curve.

3. The equation $r = 2 - 2\cos\theta$ represents a "cardioid".

   (a) Sketch the cardioid.

   (b) Find the area enclosed by the cardioid.

   (c) Compute the arc length of the cardioid.

4. Consider the sequence of circles, C_n, defined by the equations $x^2 + \left(y + \frac{1}{\sqrt{n}}\right)^2 = \frac{1}{n}$. Define a_n as the area of circle C_n and b_n as the area between circles C_n and C_{n+1}.

   (a) Sketch the picture of this infinite sequence of circles.

   (b) Does $\sum_{n=1}^{\infty} a_n$ converge?

   (c) Does $\sum_{n=1}^{\infty} b_n$ converge?

   (d) Define the circles D_n by the equations $x^2 + \left(y + \frac{1}{n}\right)^2 = \frac{1}{n^2}$ with d_n as the area of D_n. Does $\sum_{n=1}^{\infty} d_n$ converge?