MA 114 Worksheet #13: Volumes of Revolution (Shell Method)

1. Conceptual understanding of disk and shell method:

   (a) Write a general integral to compute the volume of a solid obtained by rotating the region under $y = f(x)$ over the interval $[a, b]$ about the y-axis using the method of cylindrical shells.

   (b) If you use the disk method to compute the same volume, are you integrating with respect to x or y? Why?

2. Sketch the enclosed region and use the Shell Method to calculate the volume of rotation about the y-axis.

   (a) $y = 3x - 2$, $y = 6 - x$, $x = 0$

   (b) $y = x^2$, $y = 8 - x^2$, $x = 0$, for $x \geq 0$

3. Sketch the enclosed region and use the Shell Method to calculate the volume of the solid when rotated about the x-axis.

   (a) $x = \frac{1}{4}y + 1$, $x = 3 - \frac{1}{4}y$, $y = 0$

   (b) $x = y(4 - y)$, $x = 0$

4. Use both the Shell and Disk Methods to calculate the volume obtained by rotating the region under the graph of $f(x) = 8 - x^3$ for $0 \leq x \leq 2$ about:

   (a) the x-axis

   (b) the y-axis

5. Use the Shell method to find the volume obtained by rotating the region bounded by $y = x^2 + 2$, $y = 6$, $x = 0$, and $x = 2$ about the following axes:

   (a) $x = 2$

   (b) $x = -3$

6. Find the volume of the solid obtained by rotating the following region about the y-axis.

   (a) The region bounded by $f(x) = e^x$ and the x-axis from $0 \leq x \leq 2$.

   (b) The region bounded by $f(x) = \sin(x)$ and the x-axis from $0 \leq x \leq \pi$.

   (c) The region bounded $f(x) = \ln(x)$ and the x-axis from $1 \leq x \leq 3$.