MA 114 Worksheet # 16: Review for Exam 2

1. Power, Maclaurin, and Taylor Series

   (a) Find the Maclaurin series for $\dfrac{x^2}{1+x}$.

   (b) Find the Taylor series for $\cos x$ about $a = \pi/2$.

   (c) Find the Taylor series centered at $c = 0$ of $\dfrac{2}{4-3x}$ and determine its radius of convergence.

   (d) Find the Taylor series centered at zero of the function $f(x) = \ln(x+5)$.

   (e) Find the Taylor series centered at zero of the function $g(x) = x^3 \ln(x^2+5)$.

2. Compute $T_3(x)$, the Taylor polynomial of the third order centered at $x = 0$, for $f(x) = \cos(x/\pi)$.

3. Compute $T_n(x)$, the Taylor polynomial of the nth order centered at $x = 0$, for $f(x) = e^{3x}$.

4. Let $f(x) = e^{-x}$. First compute $T_3(x)$ and then use the error bound to show that $|f(x) - T_3(x)| \leq x^4/24$ for all $x \geq 0$.

5. Density and average value:

   (a) Find the total mass of a circular plate of radius 20 cm whose mass density is the radial function $\rho(r) = 0.03 + 0.01\cos(\pi r^2)$ g/cm^2.

   (b) Find the average value of $f(x) = \sin(x)\cos(x)$ over $[0, \pi]$.

6. Volume of solid with known cross section:
   Calculate the volume of the following solid. The base is the region enclosed by $y = 2 - x^2$ and the x-axis. The cross sections perpendicular to the y-axis are squares.

7. Volumes:

   (a) (Disks) Let V be the volume of a right circular cone of height 10 whose base is a circle of radius 4. Use similar triangles to find the area of a horizontal cross section at a height y. Using this area, calculate the volume V by integrating the cross-sectional area.

   (b) (Washers) Let R be a region bounded by $y = x^2$ and $y = 1$, if R is rotated about x-axis, what is the volume of the resulting solid?

   (c) (Cylindrical Shells) V is obtained by rotating the region under the graph $y = 3x^2$ for $0 \leq x \leq 2$ about the y-axis. Calculate the volume of V.

8. Work:
   Calculate the work against gravity required to build a right circular cone of height 4 m and radius 2 m out of a lightweight material of density 600 kg/m^3. (See also question 7(a).)

9. Trigonometric Integrals:

   (a) $\displaystyle\int \sin^2(x)\cos^3(x)\,dx$

   (b) $\displaystyle\int \tan^3(x)\sec^3(x)\,dx$