MA 114 Worksheet # 18:
Method of Partial Fractions & Numerical Integration

1. Write out the general form for the partial fraction decomposition but do not determine the numerical value of the coefficients.
   (a) $\dfrac{1}{x^2+3x+2}$
   (b) $\dfrac{x+1}{x^2+4x+4}$
   (c) $\dfrac{x}{(x^2+1)(x+1)(x+2)}$
   (d) $\dfrac{2x+5}{(x^2+1)^3(2x+1)}$

2. Compute the following integrals.
   (a) $\displaystyle\int \frac{x-9}{(x+5)(x-2)}\,dx$
   (b) $\displaystyle\int \frac{1}{x^2+3x+2}\,dx$
   (c) $\displaystyle\int \frac{x^3-2x^2+1}{x^3-2x^2}\,dx$
   (d) $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx$
   (e) $\displaystyle\int \frac{1}{x(x^2+1)}\,dx$

3. Compute
$$\int \frac{1}{\sqrt{x}-\sqrt[3]{x}}\,dx$$
   by first making the substitution $u = \sqrt[6]{x}$.

4. Conceptual Understanding:
   (a) Write down the Midpoint rule and illustrate how it works with a sketch.
   (b) Write down the Trapezoidal rule and the error bound associated with it.

5. Use the Midpoint rule to approximate the value of $\displaystyle\int_{-1}^{1} e^{-x^2}\,dx$ with $n = 4$. Draw a sketch to determine if the approximation is an overestimate or an underestimate of the integral.