

MA 114 Worksheet # 2: Improper Integrals

1. For each of the following, determine if the integral is proper or improper. If it is improper, explain why. Do **not** evaluate any of the integrals.

(a) $\int_0^2 \frac{x}{x^2 - 5x + 6} dx$

(d) $\int_{-\infty}^{\infty} \frac{\sin x}{1 + x^2} dx$

(b) $\int_1^2 \frac{1}{2x - 1} dx$

(e) $\int_0^{\pi/2} \sec x dx$

(c) $\int_1^2 \ln(x - 1) dx$

2. For the integrals below, determine if the integral is convergent or divergent. Evaluate the convergent integrals.

(a) $\int_{-\infty}^0 \frac{1}{2x - 1} dx$

(c) $\int_0^2 \frac{x - 3}{2x - 3} dx$
(write the numerator as $\frac{1}{2}(2x - 3) - \frac{3}{2}$)

(b) $\int_{-\infty}^{\infty} x e^{-x^2} dx$

(d) $\int_0^{\infty} \sin \theta d\theta$

3. Consider the improper integral

$$\int_1^{\infty} \frac{1}{x^p} dx.$$

Integrate using the generic parameter p to prove the integral converges for $p > 1$ and diverges for $p \leq 1$. You will have to distinguish between the cases when $p = 1$ and $p \neq 1$ when you integrate.

4. Use the Comparison Theorem to determine whether the following integrals are convergent or divergent.

(a) $\int_1^{\infty} \frac{2 + e^{-x}}{x} dx$

(b) $\int_1^{\infty} \frac{x + 1}{\sqrt{x^6 + x}} dx$

5. Explain why the following computation is wrong and determine the correct answer. (Try sketching or graphing the integrand to see where the problem lies.)

$$\begin{aligned} \int_2^{10} \frac{1}{2x - 8} dx &= \frac{1}{2} \int_{-4}^{12} \frac{1}{u} du \\ &= \frac{1}{2} \ln |x| \Big|_{-4}^{12} \\ &= \frac{1}{2} (\ln 12 - \ln 4) \end{aligned}$$

where we used the substitution

$$\begin{cases} u(x) = 2x - 8 \\ u(2) = -4, u(10) = 12 \\ \frac{du}{dx} = 2 \end{cases}$$