MA 114 Worksheet # 28:
Arc Length, Speed, Surface Area & Polar Coordinates

1. Consider the curve parametrized by $c(t) = (t^4, t^6)$
   (a) Find a cartesian equation for this curve.
   (b) Find the arc length for this curve for $0 \leq t \leq 1$. Which part of the curve given in part (a) does this compute?
   (c) Find the arc length for this curve for $-1 \leq t \leq 1$. Which part of the curve given in part (a) does this compute? How do you interpret your answer?

2. A "logarithmic spiral" is parametrized by $c(t) = (e^t \cos(t), e^t \sin(t))$.
   (a) Find the slope of the tangent lines, and use this to sketch this curve, for $0 \leq t \leq 2\pi$.
   (b) Find the speed $s'(t)$.
   (c) Find the length of the curve, again for $0 \leq t \leq 2\pi$.
   (d) What does the curve look like, for $-2\pi \leq t \leq 0$?

3. The curve parametrized by $c(t) = (\cos^3(t), \sin^3(t))$ is known as the "astroid".
   (a) Sketch this curve, for $0 \leq t \leq \pi$.
   (b) Find the length of this curve.
   (c) Find the area of the surface obtained by revolving the astroid around the x-axis.

4. Convert from rectangular to polar coordinates:
   (a) $(1, \sqrt{3})$
   (b) $(-1, 0)$
   (c) $(2, -2)$

5. Convert from polar to rectangular coordinates:
   (a) $\left(2, \frac{\pi}{6}\right)$
   (b) $\left(-1, \frac{\pi}{2}\right)$
   (c) $\left(1, -\frac{\pi}{4}\right)$

6. Sketch the graph of the polar curves:
   (a) $\theta = \frac{3\pi}{4}$
   (b) $r = \pi$

7. Find the equation in polar coordinates of the line through the origin with slope $\frac{1}{3}$.

8. Find the polar equation for:
   (a) $x^2 + y^2 = 9$
   (b) $x = 4$
   (c) $y = 4$

9. Convert the equation of the circle $r = 2\sin\theta$ to rectangular coordinates and find the center and radius of the circle.

10. Given the circle represented by $x^2 + (y-2)^2 = 4$
    (a) Find the polar representation for this equation.
    (b) Calculate the area enclosed by $0 \leq \theta \leq \pi/4$.
    (c) Sketch the area calculated.