MA 114 Worksheet # 8: Review for Exam 1

1. Integration by Parts:

   (a) $\int x^2 \cos(x)\, dx$

   (b) $\int 2x \arctan(x)\, dx$

2. Improper Integrals:

   (a) Evaluate $\int_{-1}^{\infty} e^{-x}\, dx$

   (b) Evaluate $\int_{1}^{2} \frac{dx}{x \ln(x)}$.

   (c) Evaluate $\int_{3}^{6} \frac{x}{\sqrt{x-3}}\, dx$.

3. Determine the limit of the sequence or state that the sequence diverges.

   (a) $a_n = \sqrt{n+3} - \sqrt{n}$

   (b) $b_n = \dfrac{\cos(n)}{n}$

   (c) $c_n = \dfrac{e^n + (-3)^n}{5^n}$

   (d) $d_n = n^{1/n}$

   [HINT: Verify that $x^{1/x} = e^{\frac{\ln(x)}{x}}$ and use l'Hospital's Rule.]

4. Determine whether or not $\displaystyle\sum_{n=2}^{\infty} \left(1 - \sqrt{1 - \frac{1}{n^2}}\right)$ converges.

5. Evaluate $\displaystyle\sum_{n=3}^{\infty} \frac{1}{n(n+3)}$.

   [HINT: Use $\dfrac{1}{n(n+3)} = \dfrac{1}{3}\left(\dfrac{1}{n} - \dfrac{1}{n+3}\right)$.]

6. Find a value of N such that S_N approximates the series with an error of at most 10^{-5} where $S = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+2)(n+3)}$

7. Determine convergence or divergence.

   (a) $\displaystyle\sum_{n=0}^{\infty} 5^{-n}$.

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$.

   (c) $\displaystyle\sum_{n=1}^{\infty} n e^{-n^2}$.

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$.

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n^n}$.

8. Determine the radius of convergence of the following series.

   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n x^n}{n!}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \frac{(2x-3)^n}{n \ln(n)}$