Worksheet # 28: Indefinite Integrals and the Net Change Theorem

1. Compute the definite integral.
   (a) \[ \int_{0}^{2} (4x^5 + x^2 + 2x + 1) \, dx \]
   (b) \[ \int_{0}^{\pi/2} (\sin x + 5 \cos x) \, dx \]
   (c) \[ \int_{1}^{16} \frac{1 + \sqrt{x}}{\sqrt{x}} \, dx \]
   (d) \[ \int_{1}^{2} \sqrt{\frac{7}{x^3}} \, dx \]

2. Find the general indefinite integral.
   (a) \[ \int \frac{15}{x} \, dx \]
   (b) \[ \int \frac{x^2 - \sqrt{x}}{x} \, dx \]
   (c) \[ \int \cos(x) - \sin(x) + e^x \, dx \]
   (d) \[ \int (1 + \tan^2 \theta) \, d\theta \]
   (e) \[ \int \sin^2 y \, dy \] [Hint: Use an identity.]

3. Let the velocity of a particle traveling along the x-axis be given by \( v(t) = t^2 - 3t + 8 \). Find the displacement and distance traveled by the particle from \( t = 2 \) to \( t = 4 \) seconds.

4. The velocity of a particle traveling along the x-axis is given by \( v(t) = 3t^2 + 8t + 15 \) and the particle is initially located 5 m left of the origin. How far does the particle travel from \( t = 2 \) seconds to \( t = 3 \) seconds? After 3 seconds where is the particle with respect to the origin?

5. (MA 113 Exam IV, Problem 7, Spring 2009). A particle is traveling along a straight line so that its velocity at time \( t \) is given by \( v(t) = 4t - t^2 \) (measure in meters per second).
   (a) Graph the function \( v(t) \).
   (b) Find the total distance traveled by the particle during the time period \( 0 \leq t \leq 5 \).
   (c) Find the net distance traveled by the particle during the time period \( 0 \leq t \leq 5 \).

6. An oil storage tank ruptures and oil leaks from the tank at a rate of \( r(t) = 100e^{-0.01t} \) liters per minute. How much oil leaks out during the first hour?

7. (Similar to problem 47, p. 397). Draw the region \( R \) that lies between the y-axis and the curve \( x = 2y - y^2 \) from \( y = 0 \) to \( y = 2 \). To find the area between a continuous function \( f \) and the x-axis on the interval \([a, b] \), we just evaluate \( \int_{a}^{b} f(x) \, dx \). Use some intuition to find the area of \( R \).