

Worksheet # 29: The Substitution Rule

1. Evaluate the following indefinite integrals. Be sure to indicate any substitutions that you use.

(a) $\int \frac{4}{(1+2x)^3} dx$

(b) $\int x^2 \sqrt{x^3+1} dx$

(c) $\int \cos^4 \theta \sin \theta d\theta$

(d) $\int \frac{1}{(5t+4)^{2.7}} dt$

(e) $\int \sec 2\theta \tan 2\theta d\theta$

(f) $\int \sec^3 x \tan x dx$

(g) $\int x^2 \cdot \sqrt[3]{x^3+1} dx$

(h) $\int \frac{dx}{2x+1}$

2. Evaluate the following definite integrals. Be sure to indicate any substitutions that you use.

(a) $\int_0^7 \sqrt{4+3x} dx$

(b) $\int_{1/6}^{1/2} \csc(\pi t) \cot(\pi t) dt$

(c) $\int_0^{\pi/2} \cos x \sin(\sin x) dx$

(d) $\int_0^4 \frac{x}{\sqrt{1+2x^2}} dx$

(e) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

(f) $\int_0^\pi x \cos x^2 dx$

(g) $\int_0^3 e^x \sin(e^x) dx$

(h) $\int_1^2 \frac{e^{1/x}}{x^2} dx$

3. If f is continuous and $\int_0^9 f(x) dx = 4$ find $\int_0^3 x \cdot f(x^2) dx$.

4. If f is continuous and $\int_0^4 f(x) dx = 10$, find $\int_0^2 f(2x) dx$.

5. Identify each of the following statements as true or false. Justify your answer.

(a) If f and g are continuous on $[a, b]$, then $\int_a^b [f(x)g(x)] dx = \left(\int_a^b f(x) dx \right) \left(\int_a^b g(x) dx \right)$.

(b) If f is continuous on $[a, b]$, then $\int_a^b 5f(x) dx = 5 \int_a^b f(x) dx$.

(c) If f' is continuous on $[1, 3]$, then $\int_1^3 f'(v) dv = f(3) - f(1)$.

(d) $\int_{-1}^1 \left(x^5 - 6x^9 + \frac{\sin x}{(1+x^4)^2} \right) dx = 0$.

(e) $\int_{-5}^5 (ax^2 + bx + c) dx = 2 \int_0^5 (ax^2 + c) dx$.

(f) If f is continuous on $[a, b]$, then $\frac{d}{dx} \left(\int_a^b f(x) dx \right) = f(x)$.