Worksheet # 30: Review for Exam IV

1. Compute the derivative of the given function.
   (a) \( f(\theta) = \cos(2\theta^2 + \theta + 2) \)
   (b) \( g(u) = \ln(\sin^2 u) \)
   (c) \( h(x) = \int_{-3599}^{x} t^2 - te^{t^2+1} \, dt \)
   (d) \( r(y) = \arccos(y^3 + 1) \)

2. Compute the following definite integrals.
   (a) \( \int_{-1}^{1} e^{u+1} \, du \)
   (b) \( \int_{-2}^{2} \sqrt{4 - x^2} \, dx \)
   (c) \( \int_{1}^{9} \frac{x - 1}{\sqrt{x}} \, dx \)
   (d) \( \int_{0}^{10} |x - 5| \, dx \)
   (e) \( \int_{0}^{\pi} \sec^2(t/4) \, dt \)
   (f) \( \int_{0}^{1} xe^{-x^2} \, dx \)

3. Provide the most general antiderivative of the following functions.
   (a) \( x^4 + x^2 + x + 1000 \)
   (b) \( (3x - 2)^{20} \)
   (c) \( \sin(\ln(x)) \)

4. Use implicit differentiation to find \( \frac{dy}{dx} \).
   (a) \( x^2 + xy + y^2 = 16 \)
   (b) \( x^2 + 2xy - y^2 + x = 2 \). Also, compute \( \frac{dy}{dx}(1, 2) \)

5. If \( F(x) = \int_{3x^2+1}^{7} \cos t \, dt \) find \( F'(x) \). Justify your work carefully.

6. Suppose a bacteria colony grows at a rate of \( r(t) = 100\ln(2)2^t \) with \( t \) in hours. By how many bacteria does the population increase from time \( t = 1 \) to \( t = 3 \)?

7. Use a left Riemann sum with 4 equal subintervals to estimate the value of \( \int_{1}^{5} x^2 \, dx \). Will this estimate be larger or smaller than the actual value of definite integral? Explain.

8. A conical tank with radius 5 m and height 10 m is being filled with water at a rate of 3 m³ per minute. How fast is the water level increasing when the height is 3 m?

9. A rectangular storage container with an open top is to have a volume of 10 m³. The length of the container is twice its width. Material for the base costs $10 per square meter while material for the sides costs $6 per square meter. Find the materials cost for the cheapest possible container.

10. State the mean value theorem. Then if \( 3 \leq f'(x) \leq 5 \) for all \( x \), find the maximum possible value for \( f(8) - f(2) \).