Continuity of a Right Inverse of the Divergence Operator

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The divergence of a vector field $u = (u_1, \ldots, u_n)$, often written as div $u = \sum_{j=1}^{n} \frac{\partial u_j}{\partial x_j}$ or $\nabla \cdot u$, is a well-known quantity in vector calculus, measuring 'sinks' and 'sources' of u. In fluid dynamics, this quantity manifests itself in the compression and rarefaction of a fluid whose velocity is given by u. The incompressibility condition on such a fluid, formulated as div $u = 0$, is well-known. A more general case, div $u = f$, is naturally a PDE of interest.

Given $f \in L_0^2(\Omega)$, a right inverse of divergence can be constructed from a singular integral kernel and used to solve div $u = f$. In my talk, I present a proof (due to Ricardo G. Durán) that this right inverse is bounded from $L_0^2(\Omega)$ to $H_0^1(\Omega)^n$ using the Fourier transform and elementary techniques.