On a generalized Derivative Nonlinear Schrödinger equation

The Derivative Nonlinear Schrödinger equation (DNLS) equation $i\psi_t + \psi_{xx} + i|\psi|^2\psi_x = 0$ is a canonical equation obtained from the Hall-MHD equations in a long-wave scaling, in the context of weakly nonlinear Alfvén waves propagating along an ambient magnetic field. It has the same scaling properties as the Nonlinear Schrödinger equation with quintic power law nonlinearity (L^2-critical) that develop singularities in a finite time. It also has the property of being completely integrable by the inverse scattering transform and has soliton solutions.

In an effort to address the open question of long-time existence, we introduced recently an L^2-supercritical version of the DNLS equation by modifying the nonlinearity $|\psi|^2\psi_x$ to $|\psi|^{2\sigma}\psi_x$ ($\sigma > 1$). Numerical simulations indicate that a finite time singularity may occur, and provide a precise description of the local structure of the solution in terms of blowup rate and asymptotic profile.

The (complex valued) profile satisfies a nonlinear elliptic equation $Q_{\xi\xi} - Q + ia(\frac{1}{2\sigma}Q + \xi Q_\xi) - ibQ_\xi + i|Q|^{2\sigma}Q_\xi = 0$, where the (real) coefficients a and b depend on σ (but not on the initial condition). Using methods of asymptotic analysis, we study the deformation of the functions Q, and parameters a, b as the nonlinearity σ tends to 1. We also check our analysis against a numerical integration of the profile equation with continuation type methods. This is an ongoing work with G. Simpson and Y. Cher.