SPEAKER:

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TITLE:

The Brezis-Nirenberg Problem on S^n, in spaces of fractional dimension

ABSTRACT:

We consider the nonlinear eigenvalue problem,

$$-\Delta_{\mathbb{S}^n} u = \lambda u + |u|^{4/(n-2)} u,$$

with $u \in H_0^1(\Omega)$, where Ω is a geodesic ball in S^n. In dimension 3, this problem was considered by Bandle and Benguria. For positive radial solutions of this problem one is led to an ordinary differential equation (ODE) that still makes sense when n is a real rather than a natural number. Here we consider precisely that situation with $2 < n < 4$. Our main result is that in this case one has a positive solution if and only if $\lambda \geq -n(n-2)/4$ is such that

$$\frac{1}{4}[(2\ell_2+1)^2 - (n-1)^2] < \lambda < \frac{1}{4}[(2\ell_1+1)^2 - (n-1)^2]$$

where ℓ_1 (respectively ℓ_2) is the first positive value of ℓ for which the associated Legendre function $\mathrm{P}_\ell^{(2-n)/2}(\cos\theta_1)$ (respectively $\mathrm{P}_\ell^{(n-2)/2}(\cos\theta_1)$) vanishes.