Comments on Project 1 - Direction fields and Solution Curves
Spring 2010

Some general comments on reading IODE project 1. Experiment more! Relate the ODE to the direction field and behavior of the solution curves! I used two symbols while grading: S meaning ‘scales’, and IC meaning ‘initial conditions’.

1. SCALES: Adjust the x and y scales as needed to capture all the behavior of the ODE. Many people did not do this for problems 3 and 4 and therefore did not see the full behavior of the solutions. For \( f(x, y) = 2x(A - x) \), the x-axis must be large enough to contain 0 and \( A \). Similarly, for the next ODE \( f(x, y) = y(B - y) \), the y-axis must contain 0 and \( B \).

2. INITIAL CONDITIONS: Many people did not think about their initial conditions. You want to choose \((x_0, y_0)\) so that a characteristic solution curve is show. For example, some people took \( y_0 = 0 \) for \( y' = ky \). Of course the solution is \( y(x) = C_0 e^{kx} \) so we must have \( C_0 = 0 \). This is not interesting! Similarly, for the logistic type ODE with \( f(x, y) = y(B - y) \), there are three types of solution curves depending on \( y_0 \). you should, for all the problems, choose your initial conditions instead of clicking on some point on the graph.

3. CHOICE OF STEP SIZE \( h \): As the direction field becomes more complicated, one should vary \( h \) and take it small. This will smooth out the kinks many people found in solution curves.

4. NUMBER OF LINE SEGMENTS: You can also change the number of line segments show for the direction field in the horizontal and vertical directions.

5. RELATION TO THE EXACT SOLUTIONS: In the four cases, the ODEs can be solved exactly. Not very many people did this. Why does the solution curve of \( y' = 2x(A - x) \) look cubic? Because the solution is \( y(x) = Ax^2 - (2/3)x^3 + C \). How about ODE 4? Is is similar to the logistic ODE we studied in section 2.5. Look there and see that the graphs are similar. The exact solution is given on page 82.

6. PERIODICITY: This will be important in Lab 2.

7. ZEROS: All the direction fields considered have zeros. For ODE 3.4, these are interesting because they lie on the line \( y = -Cx \). How do they affect the solution curves? You can solve this ODE exactly and see how!

8. EXPERIMENT: This is one of the goals of using IODE. With little effort, we can change IC and parameters. This will become more important in later labs.