

Project 1 - Direction fields and Solution Curves

DUE DATE: in class, Friday, 5 February 2010

These notes basically are from the IODE project and are written by R. Laugesen of the University of Illinois.

1. GOAL OF PROJECT I

Consider the ordinary differential equation

$$(1.1) \quad \frac{dy}{dx} = f(x; y).$$

We aim to understand graphically how properties of the function $f(x, y)$ determine the direction field. In particular, we consider functions f that are everywhere positive or negative, or that depend only on x or only on y . In Project 2 we will consider functions f that are periodic in x or in y .

2. TIPS ON USING IODE

Use the menu item **Enter differential equation**, in the Direction Fields module of IODE, to enter a differential equation. Use the menu item **Change display parameters** to enter the domain and range of the plot, and then left-click on the graph to plot a solution curve. Make sure you use proper syntax when entering the ODE. See the files on Matlab and IODE syntax posted on the course web page.

3. PROJECT EXERCISES

Let k and A, B, C be fixed real constants. Consider the following 4 ODEs, each with a different characteristic:

$$(3.1) \quad \frac{dy}{dx} = ky$$

$$(3.2) \quad \frac{dy}{dx} = 2x(A - x)$$

$$(3.3) \quad \frac{dy}{dx} = y(B - y)$$

$$(3.4) \quad \frac{dy}{dx} = y + Cx$$

For each of the ODEs, complete the following three exercises:

- (1) Clearly identify the direction field for each ODE.
- (2) Choose two values for the constants in each ODE (one positive value and one negative value) and make plots of the direction fields. Each plot should also show a solution curve with your choice of the initial condition. Use the **Enter caption** menu item to add your last name, problem number, and values of the constants and initial condition to the plot.
- (3) For each direction field plotted in (1), describe in a few sentences what the distinctive features of the direction fields are. For example, for ODE (1) with $k = 2$ so $y' = 2y$, the distinctive feature is described as: *The direction field is $f(x, y) = 2y$. It is horizontal at $y = 0$, positive for $y > 0$ and negative for $y < 0$.*

- (4) To identify distinctive features, it might help to blur your vision a little when looking at the direction field. Then check that your *distinctive feature* can be justified from the form of the ODE. For example, check that the ODE $y' = 2y$ does imply that the slope of the tangent line to the solution curves is zero at $y = 0$.

Hand-in the two plots for each of the four ODEs plus a description of the direction field. PLEASE STAPLE your work.