Project 1 - Direction fields and Solution Curves

DUE DATE: in class, Wednesday, 9 February 2011

These notes basically are from the IODE project and are written by R. Laugesen of the University of Illinois.

1. Goal of Project I

Consider the ordinary differential equation

$$\frac{dy}{dx} = f(x;y). \tag{1.1}$$

We aim to understand graphically how properties of the function $f(x,y)$ determine the direction field. In particular, we consider functions f that are everywhere positive or negative, or that depend only on x or only on y. In Project 2 we will consider functions f that are periodic in x or in y.

2. Tips on using IODE

Use the menu item **Enter differential equation**, in the Direction Fields module of IODE, to enter a differential equation. Use the menu item **Change display parameters** to enter the domain and range of the plot, and then left-click on the graph to plot a solution curve. Make sure you use proper syntax when entering the ODE. See the files on Matlab and IODE syntax posted on the course web page.

3. Project Exercises

Let k and A, B, C be fixed real constants. Consider the following 4 ODEs, each with a different characteristic:

$$\frac{dy}{dx} = ky \tag{3.1}$$

$$\frac{dy}{dx} = 2x(A-x) \tag{3.2}$$

$$\frac{dy}{dx} = y(B-y) \tag{3.3}$$

$$\frac{dy}{dx} = y + Cx \tag{3.4}$$

For each of the ODEs, complete the following **three** exercises:

(1) Clearly identify the direction field for each ODE.

(2) Choose one positive and one negative value for the constants in each ODE and make plots of the direction fields (two plots for each of the four ODEs). Each plot should also show a solution curve with your choice of the initial condition. Use the **Enter caption** menu item to add your last name, problem number, and values of the constant and initial condition to the plot.

(3) For each direction field plotted in (1), describe in a few sentences what the distinctive features of the direction fields are. For example, for ODE (1) with $k = 2$ so $y' = 2y$, the distinctive feature is described as: *The direction field is $f(x, y) = 2y$. It is horizontal at $y = 0$, positive for $y > 0$ and negative for $y < 0$.*

(4) Remark: To identify distinctive features, it might help to blur your vision a little when looking at the direction field. Then check that your *distinctive feature* can be justified from the form of the ODE. For example, check that the ODE $y' = 2y$ does imply that the slope of the tangent line to the solution curves is zero at $y = 0$.

Hand-in the two plots for each of the four ODEs plus a description of the direction field. PLEASE STAPLE your work.