Project 1 - Direction fields and Solution Curves

DUE DATE: in class, Friday, 18 September 2009

These notes basically are from the IODE project and are written by R. Laugesen of the University of Illinois.

1. Goal of Project I

Consider the ordinary differential equation

$$\frac{dy}{dx} = f(x; y). \tag{1.1}$$

We aim to understand graphically how properties of the function $f(x, y)$ determine the direction field. In particular, we consider functions f that are everywhere positive or negative, or that depend only on x or only on y. In Project 2 we will consider functions f that are periodic in x or in y.

2. Tips on using IODE

Use the menu item **Enter differential equation**, in the Direction Fields module of IODE, to enter a differential equation. Use the menu item **Change display parameters** to enter the domain and range of the plot, and then left-click on the graph to plot a solution curve. Make sure you use proper syntax when entering the ODE. See the on Matlab and IODE syntax.

3. Project Exercises

Let k and A, B, C be fixed real constants. Consider the following 6 ODEs, each with a different characteristic:

$$\frac{dy}{dx} = ky, \text{ where } k > 0 \tag{3.1}$$

$$\frac{dy}{dx} = ky, \text{where } k < 0 \tag{3.2}$$

$$\frac{dy}{dx} = 2x(A - x) \tag{3.3}$$

$$\frac{dy}{dx} = y(B - y) \tag{3.4}$$

$$\frac{dy}{dx} = x^2 + Ce^{x^2}\cos^2(3y) \tag{3.5}$$

$$\frac{dy}{dx} = -e^y - C(1 - \cos(x)) \tag{3.6}$$

For each of the ODEs, complete the following three exercises:

(1) Choose two values for the constants in each ODE and make plots of the direction fields. Each plot should also show a solution curve with your choice of the initial condition. Use the **Enter caption** menu item to add your last name, problem number, and values of the constants and initial condition to the plot.

(2) For each direction field plotted in (1), describe in a few sentences what the distinctive features of the direction fields are. For example, for ODE (1) with $k = 2$ so $y' = 2y$, the distinctive feature is described as: *The direction field is horizontal at* $y = 0$.

(3) To identify distinctive features, it might help to blur your vision a little when looking at the direction field. Then check that your *distinctive feature* can be justified from the form of the ODE. For example, check that the ODE $y' = 2y$ does imply that the slope of the tangent line to the solution curves is zero at $y = 0$.

Hand-in the two plots for each of the six ODEs plus a description of the direction field. PLEASE STAPLE your work.