Comments on Project 2 - Direction fields and Solution Curves, Part 2

Some general comments on reading IODE project 2. In general, the papers were better. As always, *Experiment more*! Relate the ODE to the direction field and behavior of the solution curves!

(1) Problem 1. The condition we need is simply $f(x, y) \geq 0$. If we have the stronger conditions $f(x, y) > 0$ then $y' > 0$ for all values of x. Otherwise, when $f(x, y) = 0$, we have $y'(x) = 0$.

(2) Problem 2. If $y'(x) = f(x)$, then for any constant point a, we have

$$y(x) = \int_a^x f(s)\ ds.$$

Now if f is periodic with period P, then $f(x + P) = f(x)$. Let's see if y is periodic. This means that there is some $T > 0$ so

$$y(x+T) = \int_a^{x+T} f(s)\ ds = \int_a^x f(s)\ ds + \int_a^T f(s)\ ds = y(x) + \int_a^T f(s)\ ds.$$

So we see that if y is to be periodic, we need

$$\int_a^T f(s)\ ds = 0.$$

Taking $T = P$, this means that the integral of f over one period should vanish. If, for example, $f(x) = \cos x$, with $p = 2\pi$, this is true and y is periodic. If, however, we take $f(x) = \cos x + 1$, then f is still periodic with period 2π, but the integral is no longer zero and y is not periodic as you can easily check by integration.

(3) Problem 3. ADJUST SCALES so you see the vertically repeating pattern clearly!

(4) Problem 4. Here, remember that the direction field is a function of x only! The solution of the ODE is, as in Problem 2, is

$$y(x) = \int_0^x f(s)\ ds + C,$$

for any constant C. I wanted you to experiment and give some examples. Suppose $f(x) = 1$ for $|x| < 1$ and $f(x) = 1/\sqrt{|x|}$, for $|x| \geq 1$. Then $y(x) \to \infty$ but not f! Even easier, take $f(x) = 1$!

(5) Problem 5. You can use a logistic ODE here. This is related to problem 6. What is the difference between $f(y) = y(2 - y)$ and $f(y) = y(y - 2)$?

(6) Problem 6. Some people confused this one: You are told that $y \to 3$ as $x \to \infty$. This is similar to the logistic ODEs we studied for the population models. You need to find a function $f(y)$ so the solutions approaches 3 if $y(0) = 0$. You can use a logistic ODE as an example. For example, try $f(y) = y(3 - y)$. This has $y' = 0$ at $y = 0$ and $y = 3$. This condition is necessary.