MA 114 Worksheet # 1: Density, Average Value

1. Conceptual Understanding:

   (a) If the linear mass density of a rod at position x is given by the function $\rho(x)$, what integral should be evaluated to find the mass of the rod between points a and b?

   (b) If the radial mass density of a disk centered at the origin is given by the function $\rho(r)$, where r is the distance from the center point, what integral should be evaluated to find the mass of a disk of radius R?

   (c) Write down the equation for the average value of an integrable function $f(x)$ on $[a, b]$.

2. Find the total mass of a 1-m rod whose linear density function is $\rho(x) = 10(x+1)^{-2}$kg/m for $0 \leq x \leq 2$.

3. Find the average value of the following functions over the given interval.

   (a) $f(x) = x^3, [0, 4]$

   (b) $f(x) = x^3, [-1, 1]$

   (c) $f(x) = \cos(x), [0, \frac{\pi}{6}]$

   (d) $f(x) = \frac{1}{x^2+1}, [-1, 1]$

   (e) $f(x) = \frac{\sin \pi/x}{x^2}, [1, 2]$

   (f) $f(x) = e^{-nx}, [-1, 1]$

   (g) $f(x) = 2x^3 - 6x^2, [-1, 3]$

   (h) $f(x) = x^n$ for $n \geq 0$, [0,1]

4. Odzala National Park in the Republic of the Congo has a high density of gorillas. Suppose that the radial population density is $\rho(r) = 52(1+r^2)^{-2}$ gorillas per square kilometer, where r is the distance from a grassy clearing with a source of water. Calculate the number of gorillas within a 5-km radius of the clearing.

5. Charge is distributed along a glass tube along a glass tube of length 10 cm with linear charge density $\rho(x) = \frac{10^{-4}x}{(x^2+1)^2}$ coulombs per centimeter for $0 \leq x \leq 10$. Calculate the total charge.

6. Find the total mass of a circular plate of radius 20 cm whose mass density is the radial function $\rho(r) = 0.03 + 0.01 \cos(\pi r^2) g/cm^2$

7. (Review) Find the area between the curves $y = x^2 + 1$ and $y = \sin(\pi x) - 1$ on the interval $[1, 3]$.

8. (Review) Find the derivative of the following functions.

   (a) $f(x) = 7x^2 + 1$

   (b) $f(x) = 16 \arctan(3x^2)$

   (c) $f(x) = \ln(8x^2 + \ln(3))$

   (d) $f(x) = 3(\sqrt{(x^3)} - \sec^2(x))^3$

   (e) $f(x) = xe^{6x+\ln(4x)}$

   (f) $f(x) = 3 \sin(x) \cos(x^2)$

9. (Review) Compute the following sums

   (a) $\sum_{i=1}^{5} 3i$

   (b) $\sum_{k=3}^{6} \sin(\frac{\pi}{2} + \pi k) + 2k$