MA 114 Worksheet # 2: Volumes

1. Conceptual Understanding:

   (a) If a solid has a cross–sectional area given by the function $A(x)$, what integral should be evaluated to find the volume of the solid?

   (b) Suppose a region R in the first quadrant is bounded above by a function $f(x)$ and bounded below by a function $g(x)$. If R is rotated around the x–axis, what method should be used to find the volume of the resulting solid?

2. Let V be the volume of a right circular cone of height 10 whose base is a circle of radius 4. Use similar triangles to find the area of a horizontal cross section at a height y. Using this area, calculate V by integrating the cross-sectional area.

3. Calculate the volume of the following solid. The base is a square, one of whose sides is the interval $[0, l]$ along the x-axis. The cross sections perpendicular to the x-axis are rectangles of height $f(x) = x^2$.

4. Calculate the volume of the following solid. The base is the region enclosed by $y = x^2$ and $y = 3$. The cross sections perpendicular to the y-axis are squares.

5. Describe the solid given by the integral

$$\pi \int_0^8 (\sqrt{x} + 1)^2 - 1 dx.$$

6. For each of the following, use disks or washers to find the an integral expression for the volume of the region. Evaluate the integrals for parts (a) and (d).

   (a) R is region bounded by $y = 1 - x^2$ and $y = 0$; about the x-axis.

   (b) R is region bounded by $y = \frac{1}{x}$, $x = 1$, $x = 2$, and $y = 0$; about the x-axis.

   (c) R is region bounded by $x = 2\sqrt{y}$, $x = 0$, and $y = 9$; about the y-axis.

   (d) R is region bounded by $y = 1 - x^2$ and $y = 0$; about $y = -1$.

   (e) Between the regions in part (a) and part (d), which volume is bigger? Why?

   (f) R is region bounded by $y = e^{-x}$, $y = 1$, and $x = 2$; about $y = 2$.

   (g) R is region bounded by $y = x$ and $y = \sqrt{x}$; about $x = 2$.

7. Find the cone obtained by rotating the region under the segment joining $(0, h)$ and $(r, 0)$ about the y-axis.

8. The torus is the solid obtained by rotating the circle $(x-a)^2+y^2 = b^2$ around the y-axis (assume that $a > b$). Show that it has volume $2\pi^2 ab^2$. (Hint: Draw a picture, set up the problem, and evaluate the integral by interpreting it as the area of a circle.)